Italian Covid-19 epidemic @ 16 March 2020: logistic and Gompertz

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Update **16 March 2020**; time evolution looks slowly saturating on all variables, with the usual uncertainties on saturation predictions. I compare logistic, Gompertz and their derivatives in the hope of better locating "the peak"; also, I look at an estimate of a "lethality density of states".

I report time evolution estimates of the Covid-19 italian epidemic using nation-wide data up to March 16 (Protezione Civile, `https://bit.ly/2UbpPzt`). The previous reports are at `https://bit.ly/2W6vs4u` as well as in posts at `https://bit.ly/2QaFQEy`.

The early-stage onset of saturated behavior is quite settled for all quantities (the simple exponential is statistically rejected). I compare the fits with logistic and Gompertz function $f(t, A, B, C) = \exp(A - C \exp(-Bt))$. An example is in Fig.1 for the death count. Sad though it is, today's data are essentially in the foreseen trend.

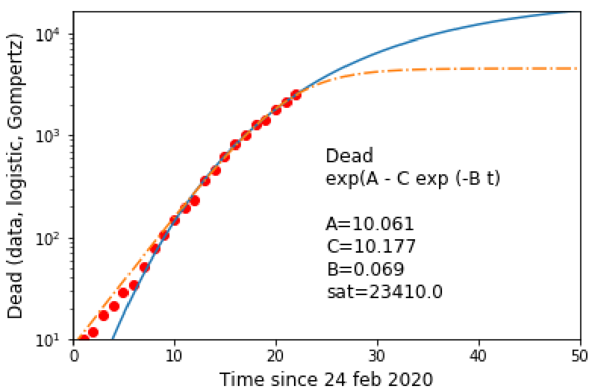

FIG. 1. Gompertz and logistic fit to the death count up to March 15, 2020. The fit parameters are reported.

Table I reports the current asymptotic values and inflection points (calculated both as zeros of second derivatives and maxima of first derivatives) together with yesterday's values. Saturations are slightly worse, aside from the slightly less insane Gompertz total of 1.3 million vs 3.5 million yesterday. Inflection times are stable for the logistic, decreasing slowly for Gompertz, but still very large. Extrapolated final lethality and death rate for hospitalized patients are currently still large at 4 and 24%.

The data at this time are compatible with both logistic and Gompertz, and we cannot yet abandon eitherhypothesis (as also borne out by Table II). The transition is probably going to be visually smooth in the cumulative distribution, as the fits adapt progressively with attendant changes in saturations and inflections; the hope is the Gompertz rate may finally decide to drop dead and bring down the long-term numbers to non-nonsensical values (although current rates are slightly worse than yesterday).

TABLE I. Predictions with logistic and Gompertz for ICU+deaths, deaths, hospitalized, total infected (data 15 March 2020). The two values per entry are for yesterday and today.

	ICU+deaths	Deaths	Hospitalized	Totali
	Saturation			
L	6859/7732	3283/4118	13678/16505	73250/67811
G	54269/54283	19870/26375	33553/44873	3805000/1383873
	Inflection (days from 24 Feb)			
L	20/21	20/21	17/19	23/23
G	39/39	32/35	23/26	71/57

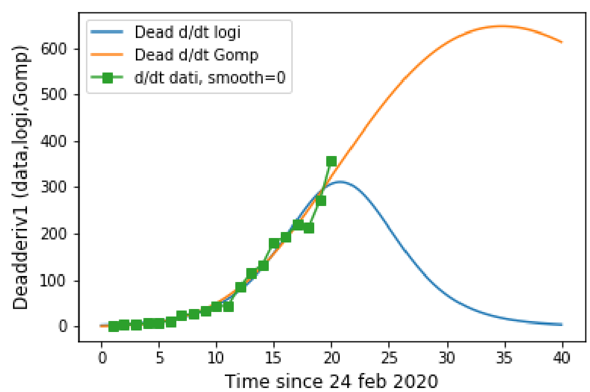

FIG. 2. Derivatives of logistic and Gompertz fitted to deaths, along with the two-point derivative of the data.

Now for a more detailed, and more pessimistic, view. Derivatives are known to blow up features of a function that are difficult to catch in the function itself (for example, derivative spectroscopies are used commonly in solid-state physics to extract fine details of the band structure). As just mentioned the cumulative curve (Fig.1) does not show yet a drastic change from logistic to Gompertz. With the usual large uncertainties, one could then look at the derivatives of Gompertz, logistic and data to better guess which one actually is "The Peak" among them. Fig.2 shows these derivatives for death count (roughly representative of other quantities too). Now, today's point could be abnormal and above the logistic peak for all sorts of reasons, but it could also be that the data *are* indeed overshooting the logistic peak and going along the Gompertz peak.

A closing positive note concerns relative lethality by

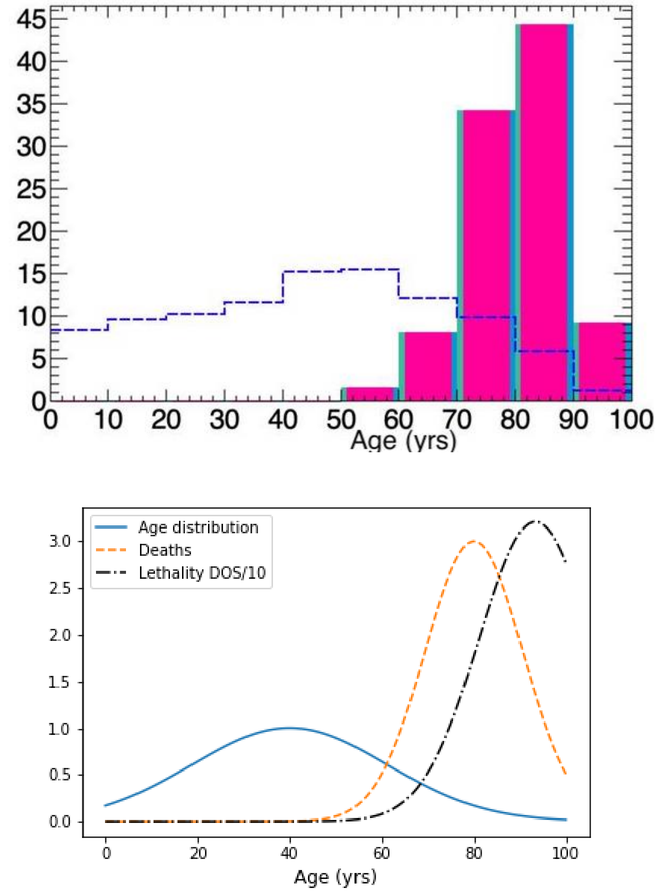

FIG. 3. Covid-related deaths by age vs Italian age distribution in % (top) and a model (bottom) of deaths by age, Italian age distribution, and lethality density of states, divided by 10.

age as observed in the national data. The graph reproduced in Fig.3 (top), courtesy of N. Tomassetti at `https://bit.ly/39WPYsm`, shows a deaths distribution very strongly skewed towards old age compared with the general population age distribution. The relevant analogy here is that of semiconductor physics: the number of carriers at a band edge vs energy is the death distribution vs age, and the Fermi distribution function is the age distribution; so we can extract a lethality "density of states" vs age simply as their ratio.

Fig.3 (bottom) shows a semiquantitative model of shape and relative weight of the three distributions (using gaussians, not a great approximation, but a handy one). The result is that at 60 (to mention a case that concerns me personally) the lethality is 35 times smaller than at the peak (around 80 yrs), and much less at younger ages. While deaths can in principle occur at any age in special situations, the risk for elderly people is huge compared to that of the younger population. This lends support to the idea of preferential quarantine for the aging, especially if ill. In passing, all this fits with the finding (`https://bit.ly/39S2JnX`) that deaths labeled as Covid-related, but with serious comorbility or debilitated general condition due to age or previous illness, are overwhelmingly more numerous that those strictly due to Covid (that currently number in the low single-digits), and the idea that these probably belong into distinct cathegories.

TABLE II. Statistical comparison (16 March 2020) for logistic (L) and Gompertz (G).

	S-p	paired-S-p	χ^2	χ^2-p	R^2
			Deaths		
L	0.989	0.612	11.783	0.945	0.999
G	0.989	0.559	32.721	0.049	0.999
			ICU+deaths		
L	0.988	0.34	24.127	0.287	1.0
G	0.996	0.658	10.827	0.966	1.0
			Hospitalized		
L	0.976	0.41	223.457	0.0	0.997
G	1.0	0.988	58.006	0.0	0.999
			Total		
L	0.99	0.49	247.642	0.0	0.999
G	1.0	0.997	62.754	0.0	0.999