SQUEEZING

Field quadratures

$$\mathcal{E}_x(z,t) = \mathcal{E}_0 \sin kz \sin(\omega t + \phi)$$

$$= \mathcal{E}_0 \sin kz (\cos\phi \sin\omega t + \sin\phi \cos\omega t)$$

$$= \mathcal{E}_1 \sin\omega t + \mathcal{E}_2 \cos\omega t$$

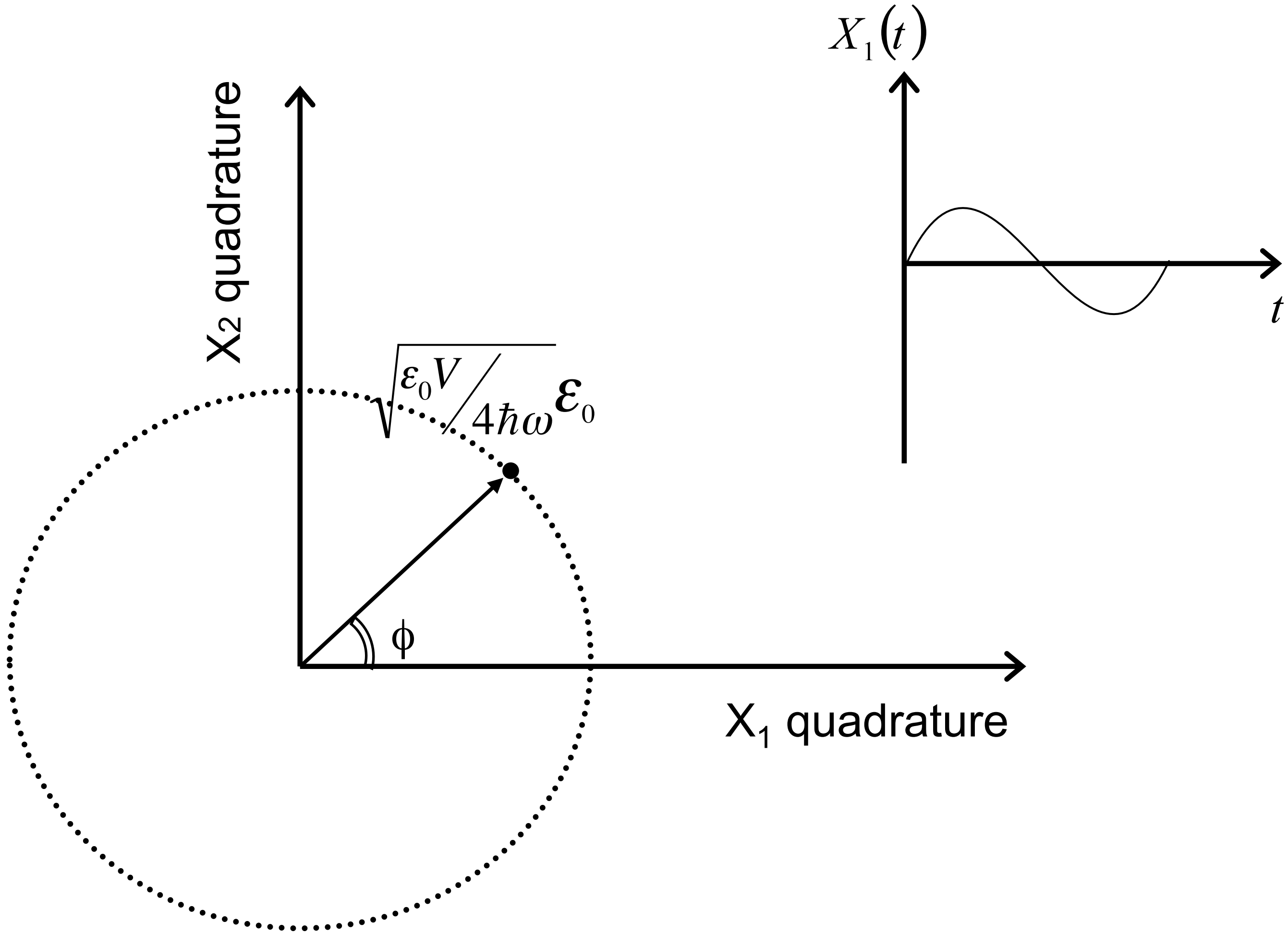
$$\mathcal{E}_1 = \mathcal{E}_0 \sin kx \cos\phi; \quad \mathcal{E}_2 = \mathcal{E}_0 \sin kx \sin\phi$$

$$X_1(t) = \sqrt{\frac{\varepsilon_0 V}{4\hbar\omega}} \mathcal{E}_0 \sin\omega t$$

$$X_2(t) = \sqrt{\frac{\varepsilon_0 V}{4\hbar\omega}} \mathcal{E}_0 \cos\omega t$$

$$\mathcal{E}_x(z,t) = \sqrt{\frac{4\hbar\omega}{\varepsilon_0 V}} \sin kx (\cos\phi X_1(t) + \sin\phi X_2(t))$$

Phasor representation



Reminder: Light waves as harmonic oscillators

New coordinates

$$q(t) = \sqrt{\frac{\varepsilon_0 V}{2\omega^2}} \mathcal{E}_0 \sin \omega t$$

$$p(t) = \sqrt{\frac{V}{2\mu_0}} B_0 \cos \omega t = \sqrt{\frac{\varepsilon_0 V}{2}} \mathcal{E}_0 \cos \omega t$$

$$p = \dot{q}$$

$$\ddot{q} = \dot{p} = -\omega^2 q$$

$$E_{em} = \frac{1}{2}\left(p^2 + \omega^2 q\right)$$

$$q(t) = \sqrt{m}\, x(t)$$

$$X_1(t) = \sqrt{\frac{\omega}{2\hbar}} q(t) \qquad X_2(t) = \sqrt{\frac{1}{2\hbar\omega}} p(t)$$

$$p(t) = \frac{1}{\sqrt{m}} p_x(t)$$

Field Quadratures

$$X_1(t) = \sqrt{\frac{\omega}{2\hbar}}q(t) \qquad X_2(t) = \sqrt{\frac{1}{2\hbar\omega}}p(t)$$

$$\hat{q}(t) = \sqrt{\frac{\hbar}{2\omega}}\left(\hat{a} + \hat{a}^{\dagger}\right)$$

$$\hat{p}(t) = \sqrt{\frac{1}{2\hbar\omega}}\left(\hat{a} - \hat{a}^{\dagger}\right)$$

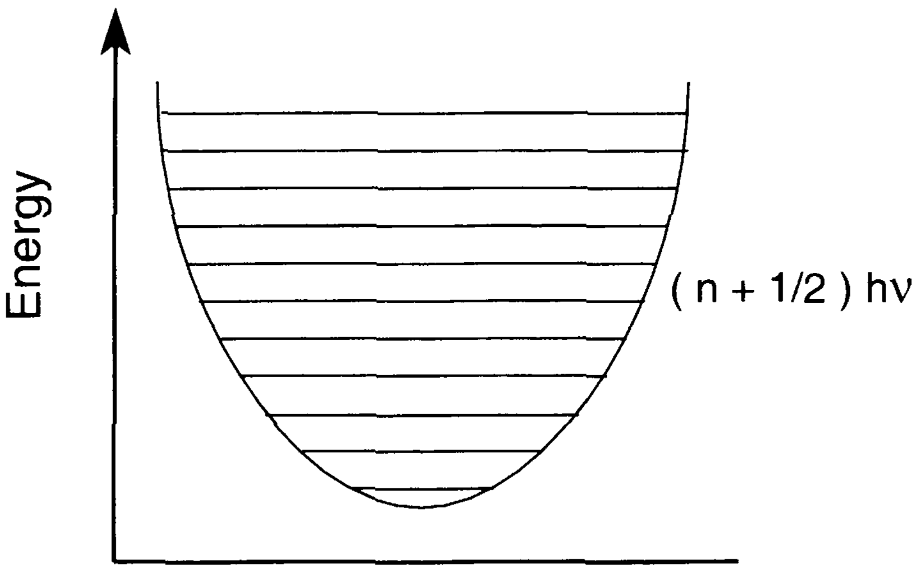
$$\hat{X}_1(t) = \frac{1}{2}\left(\hat{a} + \hat{a}^{\dagger}\right) \qquad \hat{X}_2(t) = \frac{1}{2}\left(\hat{a} - \hat{a}^{\dagger}\right)$$

Uncertainty on field quadratures

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega$$

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \qquad \Rightarrow \qquad \Delta q \Delta p \geq \frac{\hbar}{2}$$

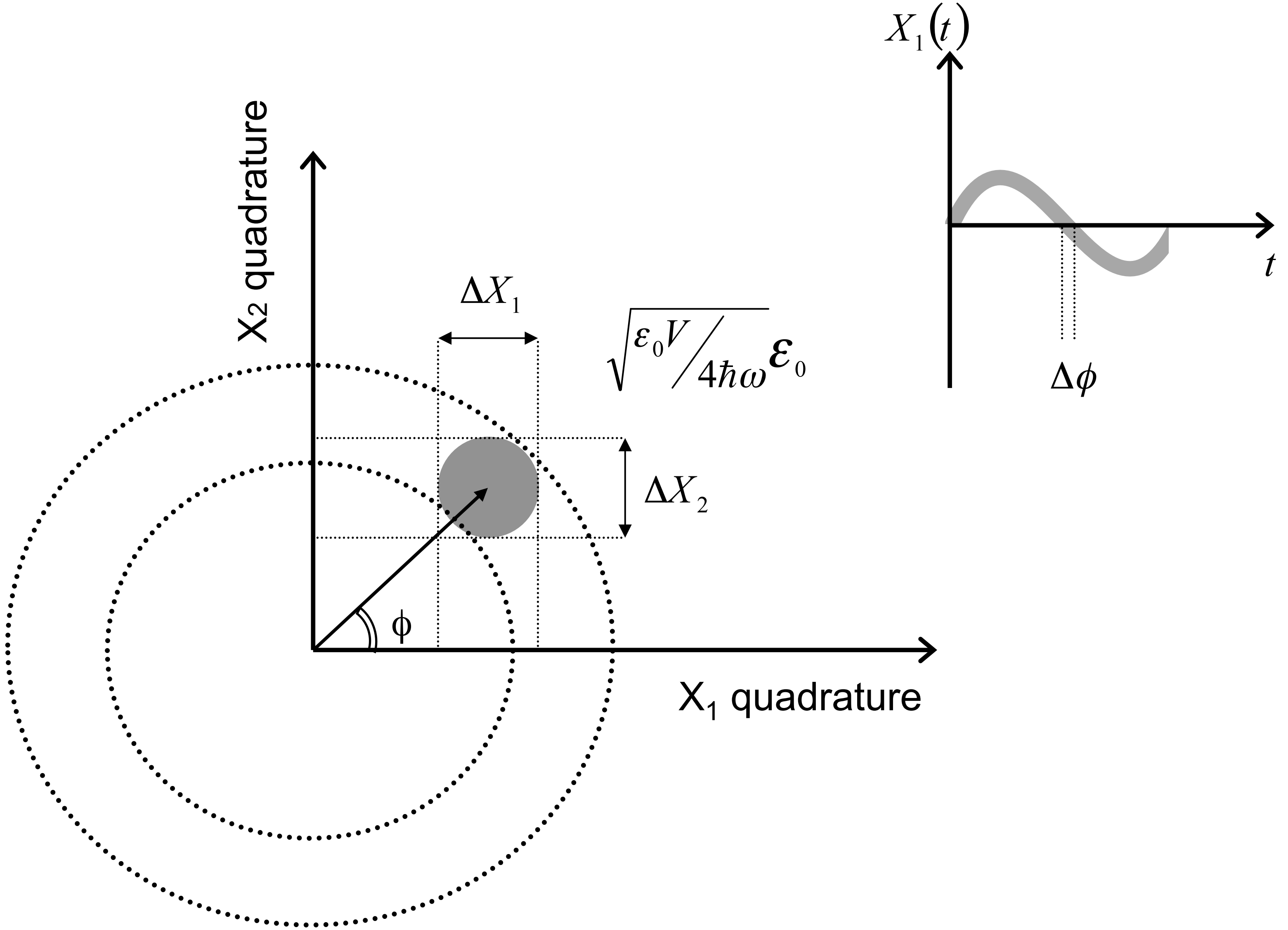
$$\Delta X_1 \Delta X_2 = \sqrt{\frac{\omega}{2\hbar}}\Delta q \sqrt{\frac{1}{2\hbar\omega}}\Delta p = \frac{1}{2\hbar}\Delta q \Delta p \geq \frac{1}{4}$$



$$q(t) = \sqrt{m}\,x(t)$$

$$p(t) = \frac{1}{\sqrt{m}}\,p_x(t)$$

Uncertainty in phasor representation



Coherent states, shot noise and number-phase uncertainty

$$\alpha = X_1 + iX_2 = |\alpha| e^{i\phi} \quad \left(\text{with } |\alpha| = \sqrt{X_1^2 + X_2^2}\right)$$

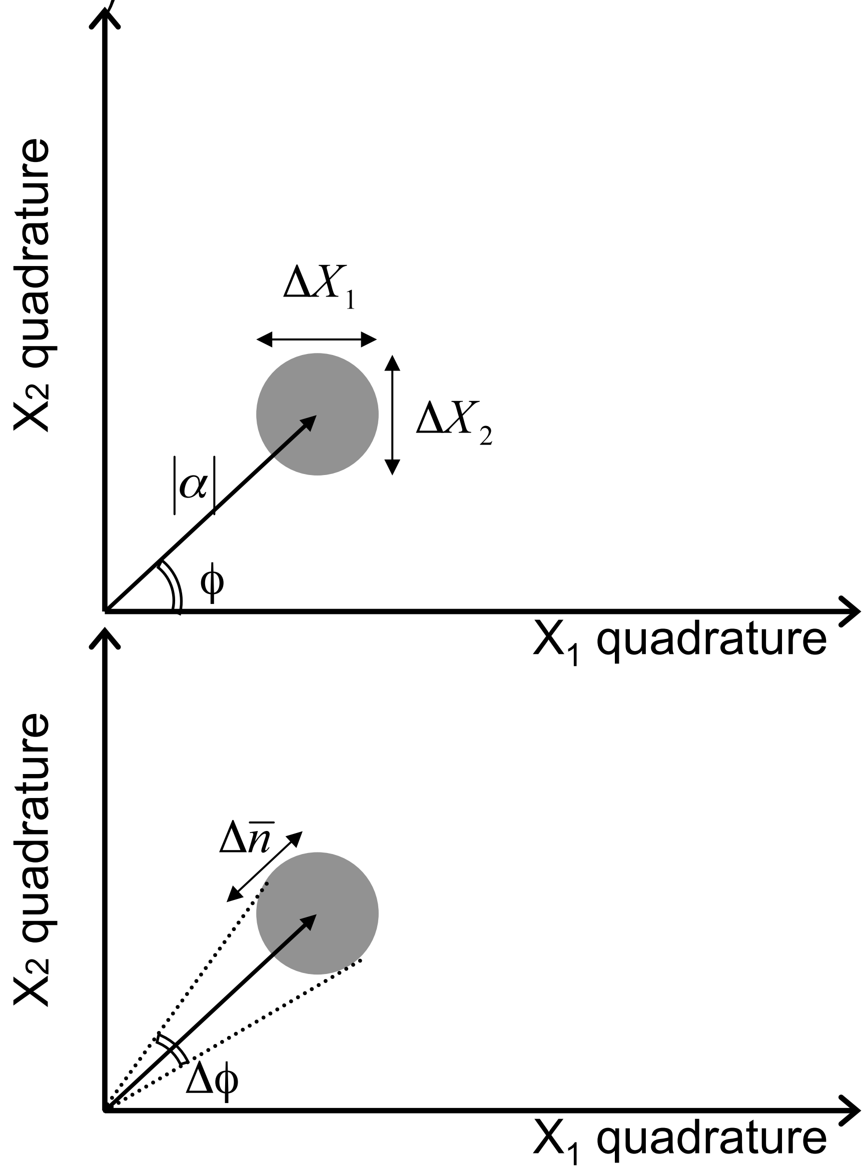
$$\Delta X_1 = \Delta X_2 = \frac{1}{2}$$

$$|\alpha|^2 = \bar{n}$$

$$\Delta n = \left(|\alpha| + \frac{1}{4}\right)^2 - \left(|\alpha| - \frac{1}{4}\right)^2 = |\alpha| = \sqrt{\bar{n}}$$

$$\Delta\phi = \frac{\text{uncertainty diameter}}{\alpha} = \frac{1/2}{\sqrt{\bar{n}}}$$

$$\Delta n \Delta\phi \geq \frac{1}{2}$$



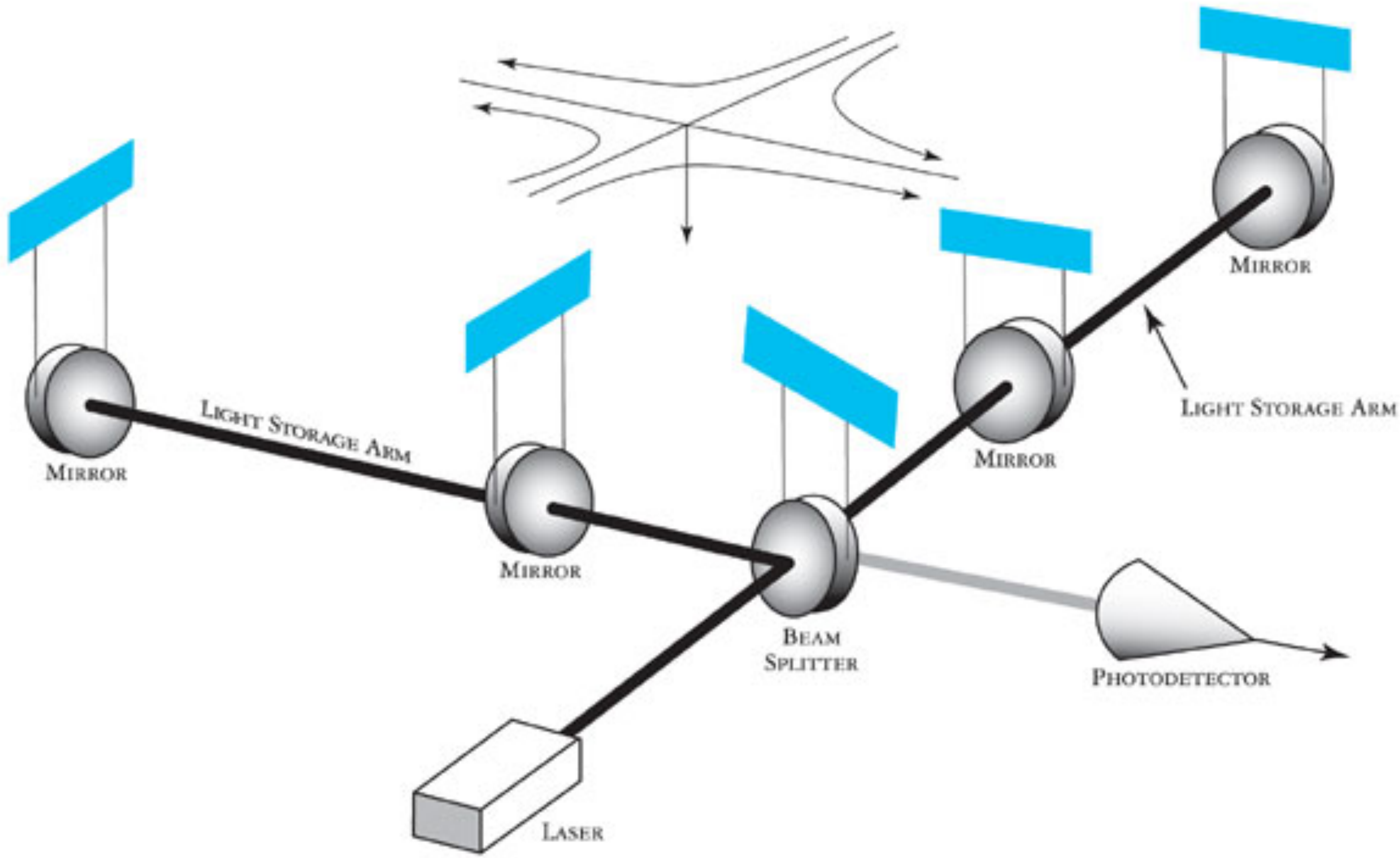
LIGO
Laser Interferometer Gravitational Wave Observatory

http://www.ligo.caltech.edu/

LIGO

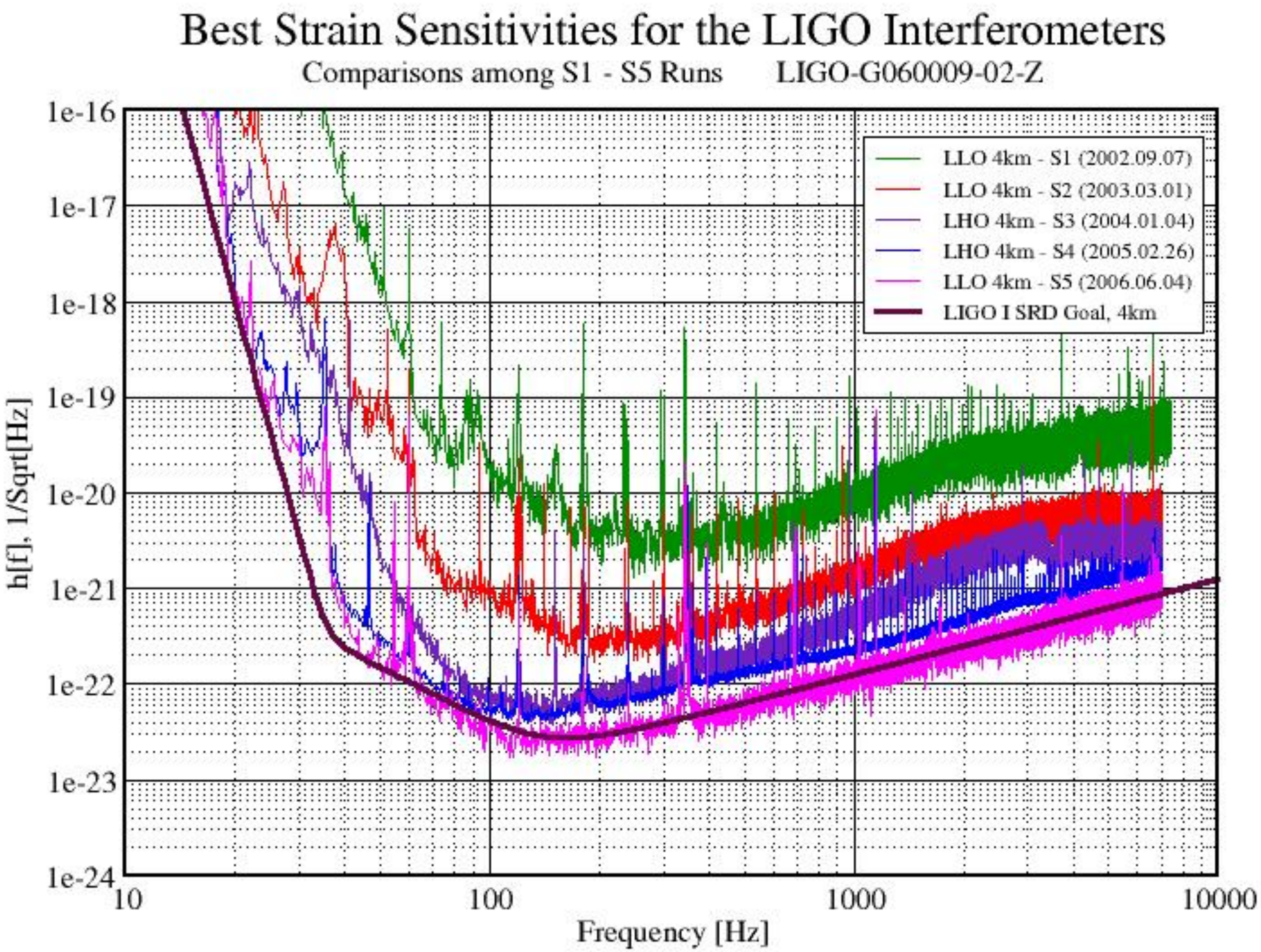
http://www.ligo.caltech.edu/

LIGO



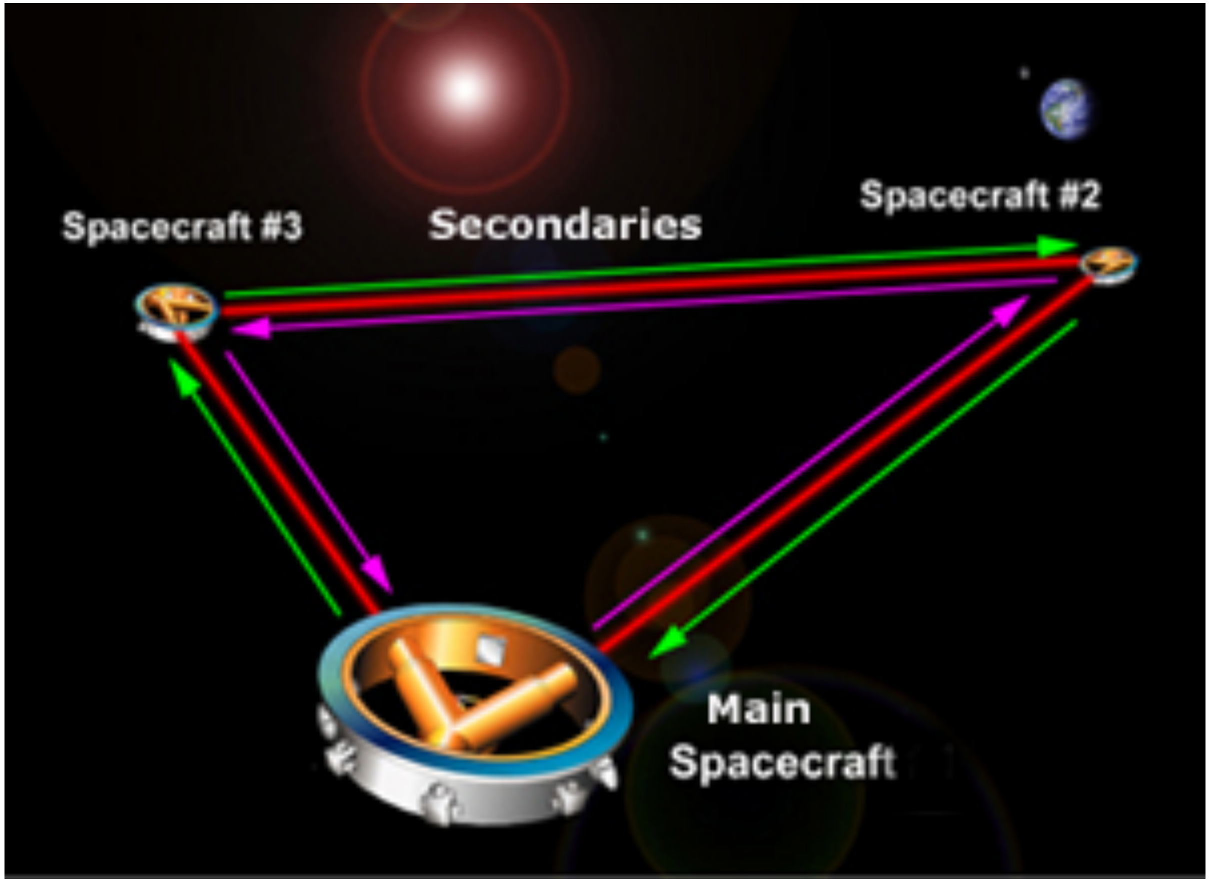
http://www.ligo.caltech.edu/

LIGO



http://www.ligo.caltech.edu/

LISA

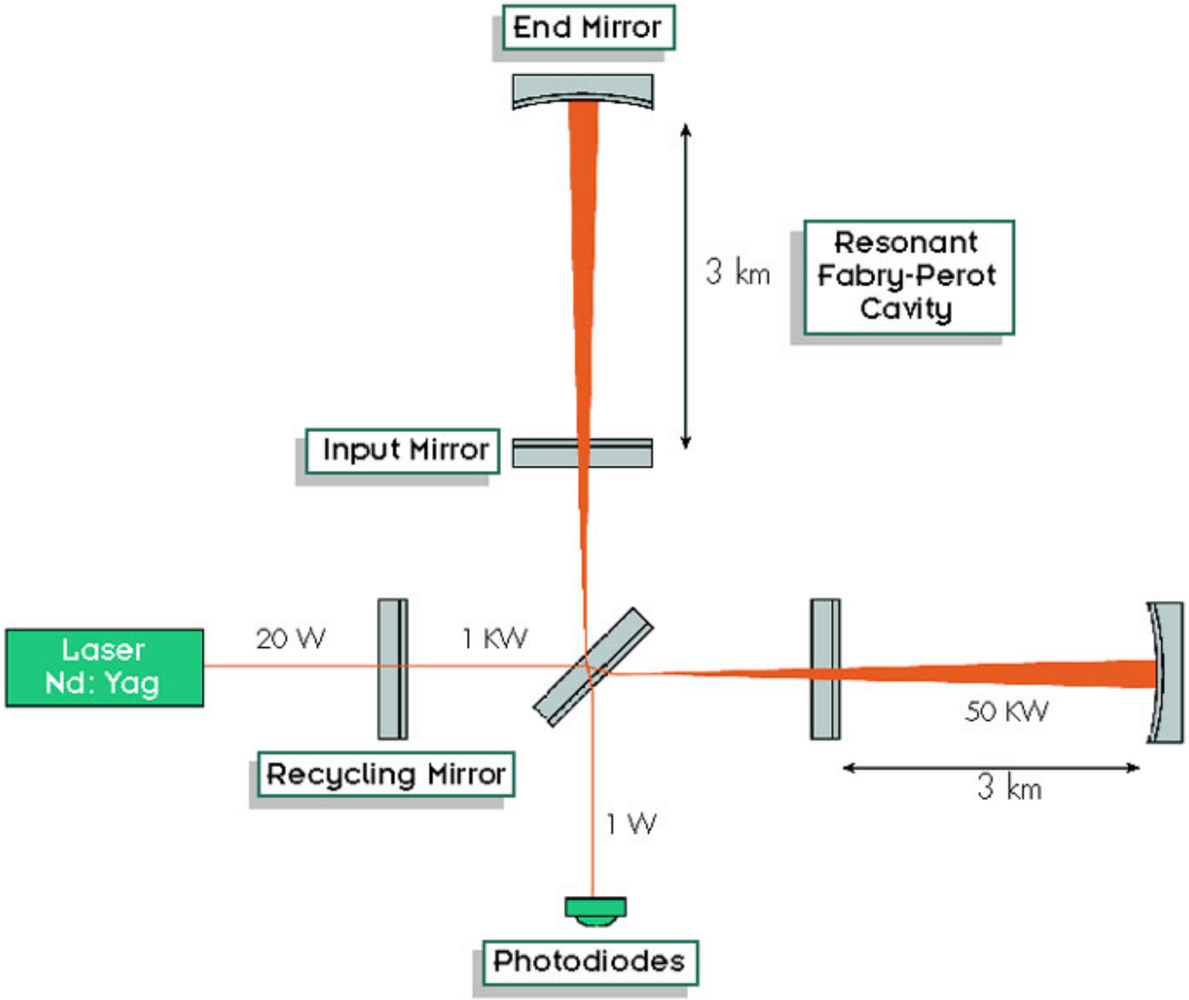


http://lisa.nasa.gov

VIRGO

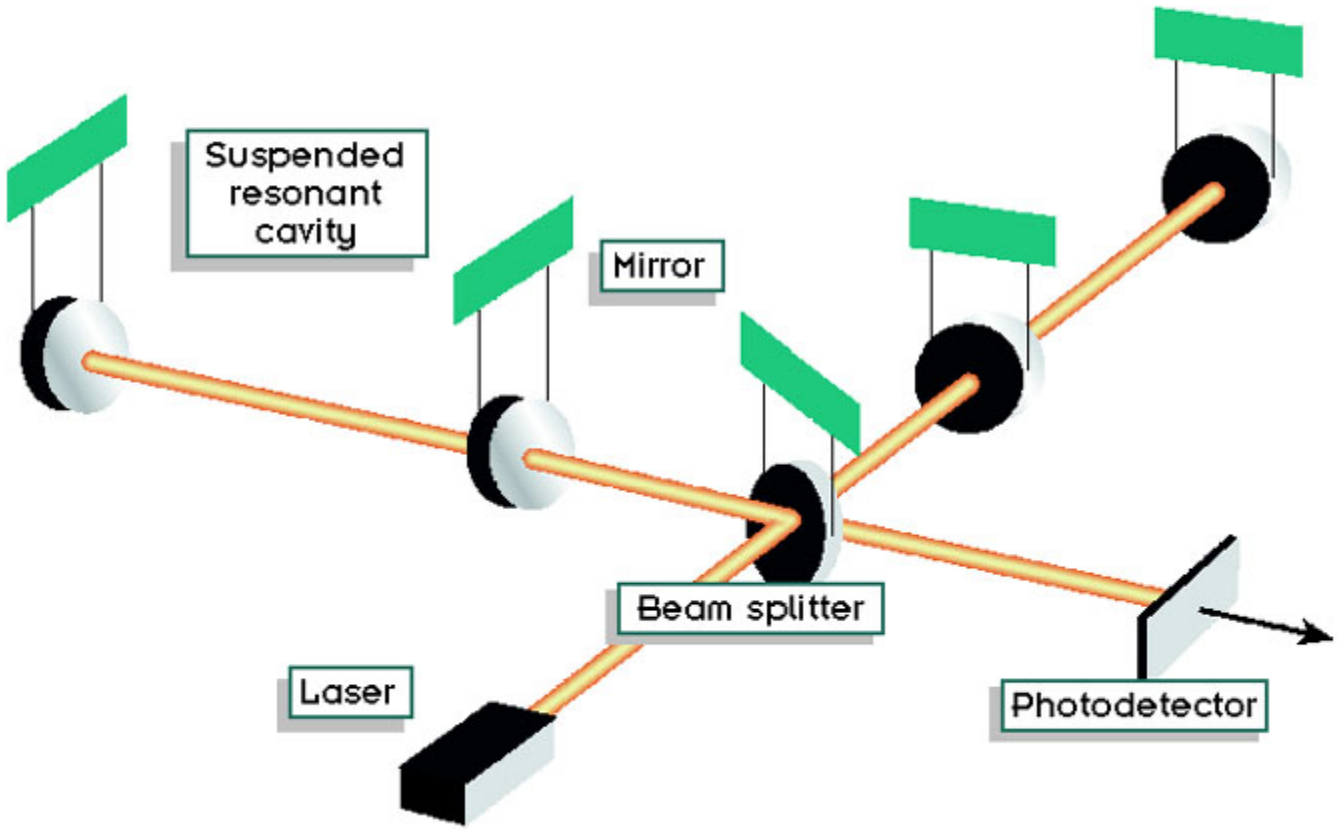
http://www.virgo.infn.it/

VIRGO



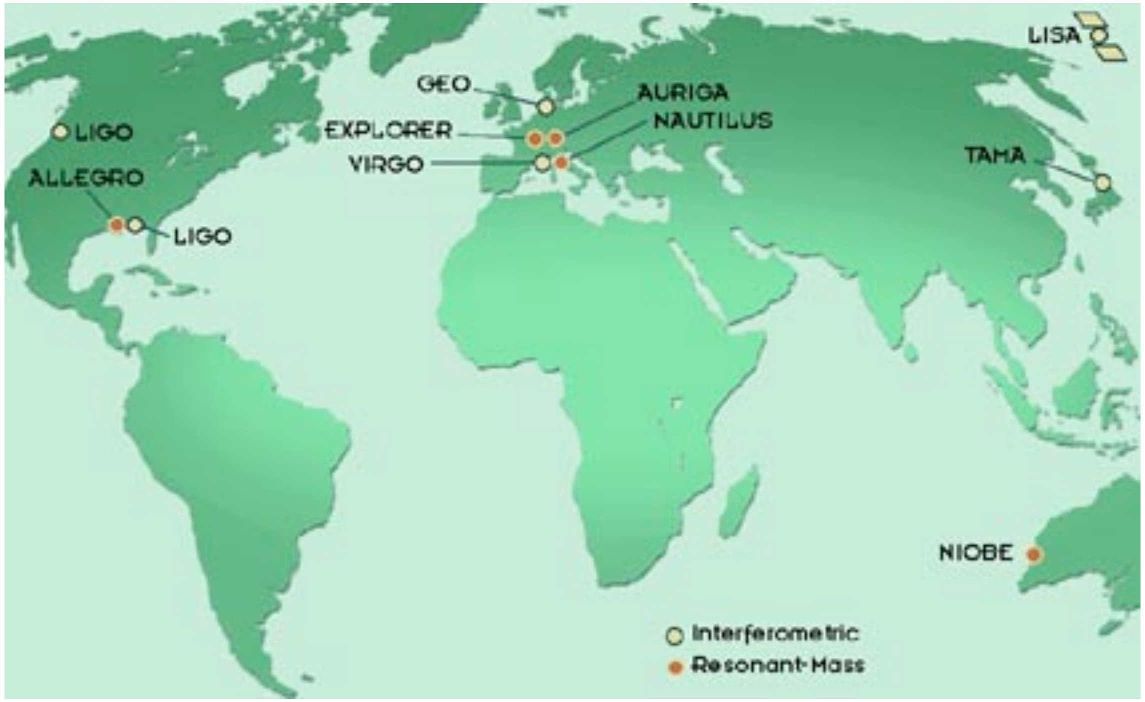
http://www.virgo.infn.it/

VIRGO



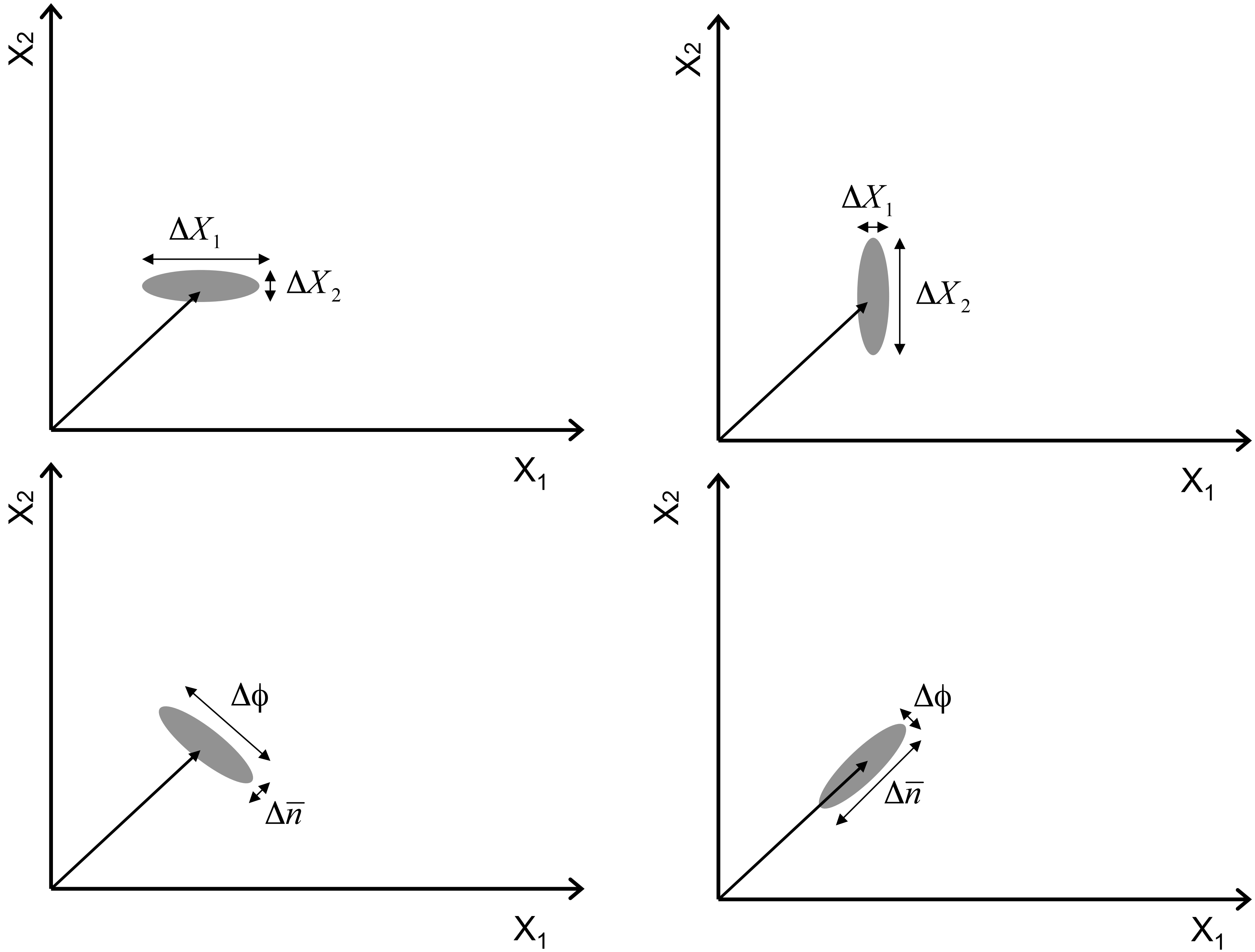
http://www.virgo.infn.it/

VIRGO

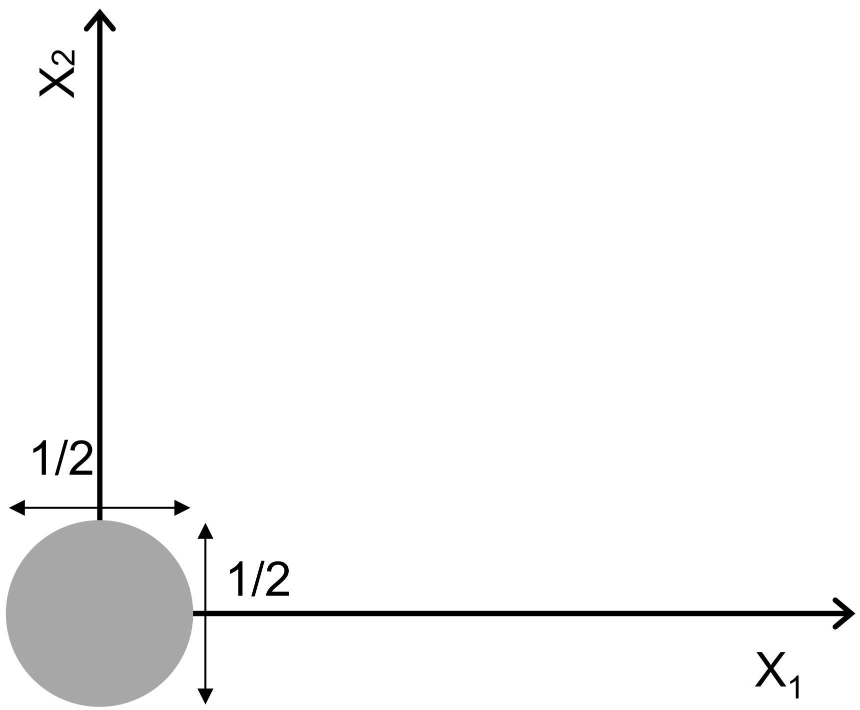


http://www.virgo.infn.it/

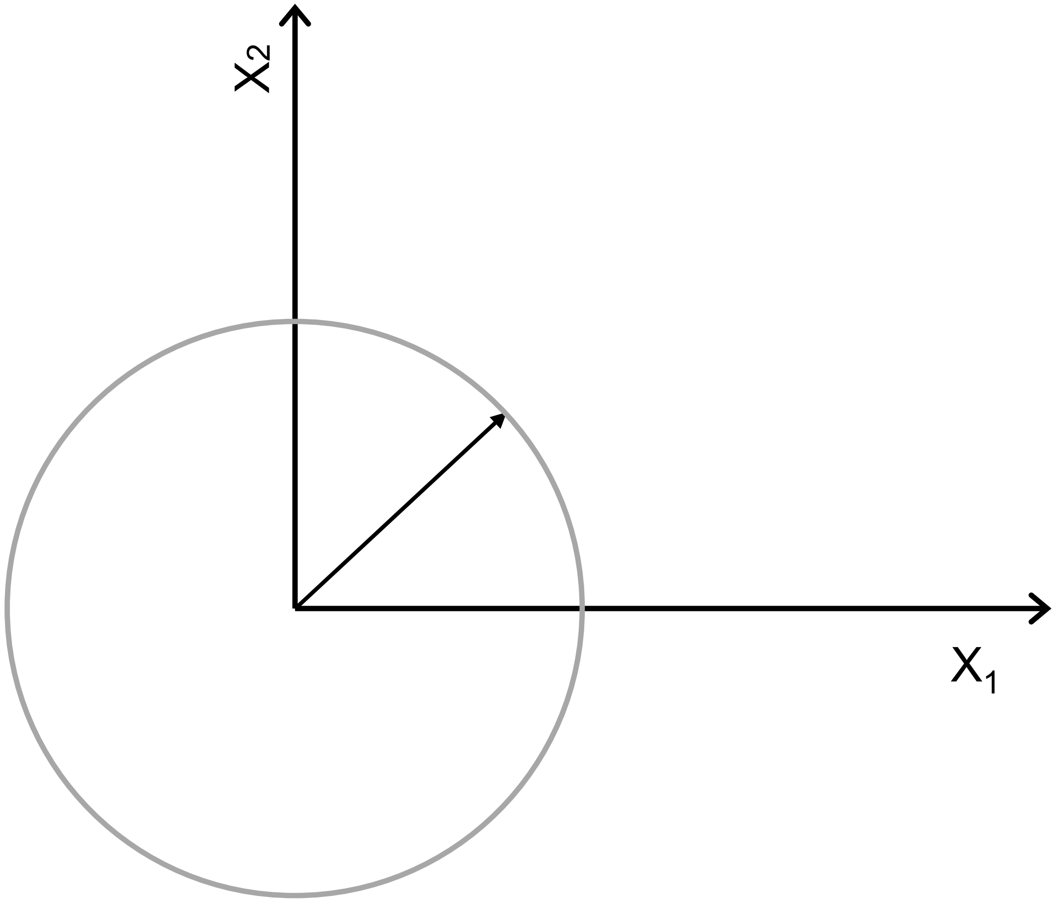
Squeezing the uncertainty



Vacuum



Number states



Squeezing in classical harmonic oscillator

$$F(x) = -kx(1-\varepsilon\sin 2\omega_0 t), \qquad \omega_0 = \sqrt{\frac{k}{m}}$$

Modulated force

$$U(x) = \frac{1}{2}kx^2(1-\varepsilon\sin 2\omega_0 t)$$

Potential energy

$$\ddot{x} + \omega_0^2 x = -\omega_0^2 x\varepsilon\sin 2\omega_0 t$$

Equation of motion

$$x(t) = B(t)\cos\omega_0 t + C(t)\sin\omega_0 t$$

$$\ddot{x}(t) = -\omega_0^2 x(t) - \omega_0\dot{B}(t)\sin\omega_0 t + \omega_0\dot{C}(t)\cos\omega_0 t$$

Trial solution and its 2nd derivative (B,C slowly varying, B''=C''=0)

Substitute into equation of motion

$$-\omega_0\dot{B}(t)\sin\omega_0 t + \omega_0\dot{C}(t)\cos\omega_0 t = -\omega_0^2\varepsilon\sin 2\omega_0 t\left[B(t)\cos\omega_0 t + C(t)\sin\omega_0 t\right]$$

Apply trigonometry to find this

$$-\dot{B}(t)\sin\omega_0 t + \dot{C}(t)\cos\omega_0 t = -\frac{\omega_0\varepsilon}{2}\left[B(t)\sin\omega_0 t + C(t)\cos\omega_0 t\right]$$

$$\dot{B}(t) = +\frac{\omega_0\varepsilon}{2}B(t); \qquad B(t) = B_0 e^{+\frac{\varepsilon\omega_0 t}{2}}$$

$$\dot{C}(t) = -\frac{\omega_0\varepsilon}{2}C(t); \qquad C(t) = C_0 e^{-\frac{\varepsilon\omega_0 t}{2}}$$

Equate sin and cos components

Displacement operator and creation of coherent states

$$\hat{D}(\alpha) = e^{\alpha \hat{a}^{+} - \alpha^{*} \hat{a}}$$

$$\hat{D}^{-1}(\alpha)\hat{a}\hat{D}(\alpha) = \hat{a} + \alpha$$

$$\hat{D}^{-1}(\alpha)\hat{a}\hat{D}(\alpha)\hat{D}^{-1}(\alpha)|\alpha\rangle = \hat{D}^{-1}(\alpha)\hat{a}|\alpha\rangle = \alpha\hat{D}^{-1}(\alpha)|\alpha\rangle$$

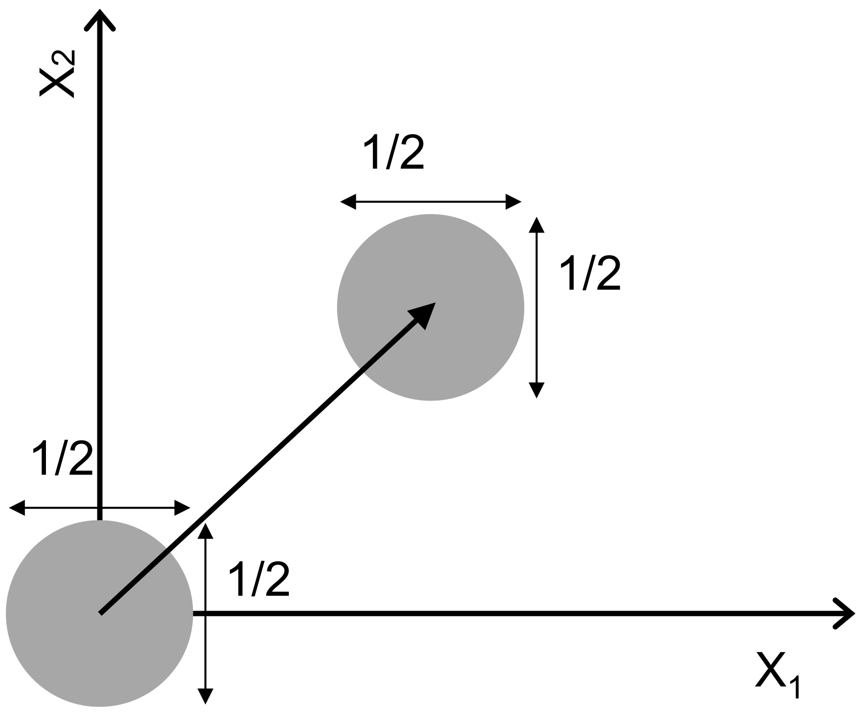
$$= (\hat{a} + \alpha)\hat{D}^{-1}(\alpha)|\alpha\rangle$$

$$\hat{a}\hat{D}^{-1}(\alpha)|\alpha\rangle = 0$$

$$\hat{D}^{-1}(\alpha)|\alpha\rangle = |0\rangle$$

$$\hat{D}(\alpha)|0\rangle = |\alpha\rangle$$

Displacement of Vacuum



Squeezing operator

$$\hat{S}(z) = e^{\frac{1}{2}\left(z\hat{a}^2 - z^*\hat{a}^{+2}\right)} \qquad z = re^{i\varphi}$$

$$\hat{t} = \hat{S}(z)\hat{a}\hat{S}^+(z) = \hat{a} + z^*\hat{a}^+ + \frac{1}{2!}|z|^2\hat{a} + \frac{1}{3!}|z|^2 z^*\hat{a}^+ + \frac{1}{4!}|z|^4\hat{a} + \ldots$$

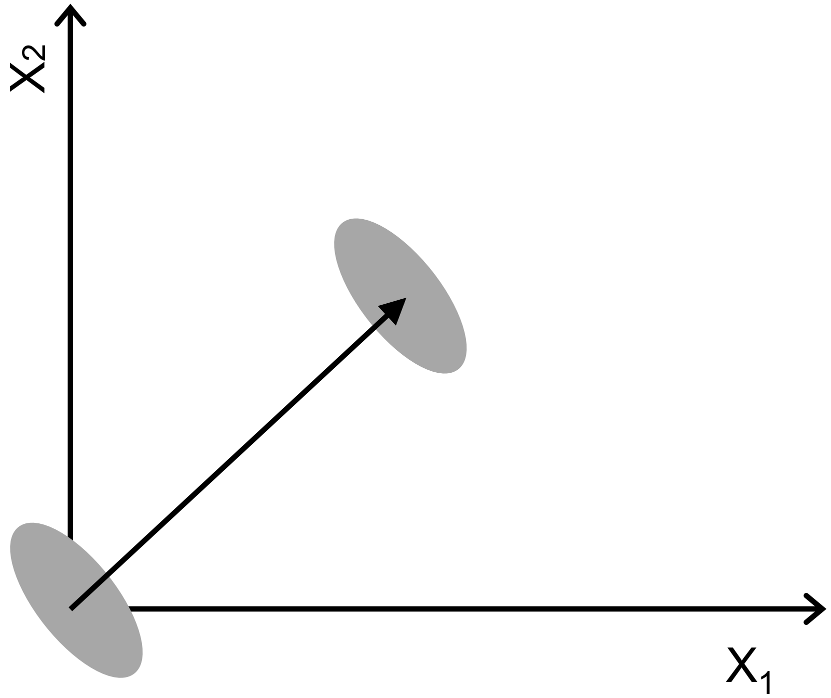
$$= \hat{a}\left(1 + \frac{1}{2!}r^2 + \frac{1}{4!}r^4 + \ldots\right) + \hat{a}^+ e^{i\varphi}\left(r + \frac{1}{3!}r^3 + \frac{1}{5!}r^5 + \ldots\right)$$

$$\hat{t} = \hat{a}\cosh r + \hat{a}^+ e^{i\varphi}\sinh r$$

$$\hat{t}\hat{S}(z)|0\rangle = 0$$

$$\hat{t}\hat{S}(z)|\alpha\rangle = tS(z)|\alpha\rangle \qquad \hat{S}(z)|\alpha\rangle = \hat{S}(z)\hat{D}(\alpha)|0\rangle$$

Displacement of Squeezed Vacuum



Squeezing operator

$$\hat{t} = \hat{a}\cosh r + \hat{a}^{+} e^{i\varphi} \sinh r$$

$$\hat{t}_1 = \frac{1}{\sqrt{2}}\left(\hat{t}^{+} + \hat{t}\right) = \frac{1}{\sqrt{2}}\left(\hat{a}^{+} + \hat{a}\right)\left(\cosh r + \sinh r\right) = \hat{X}_1 e^{r}$$

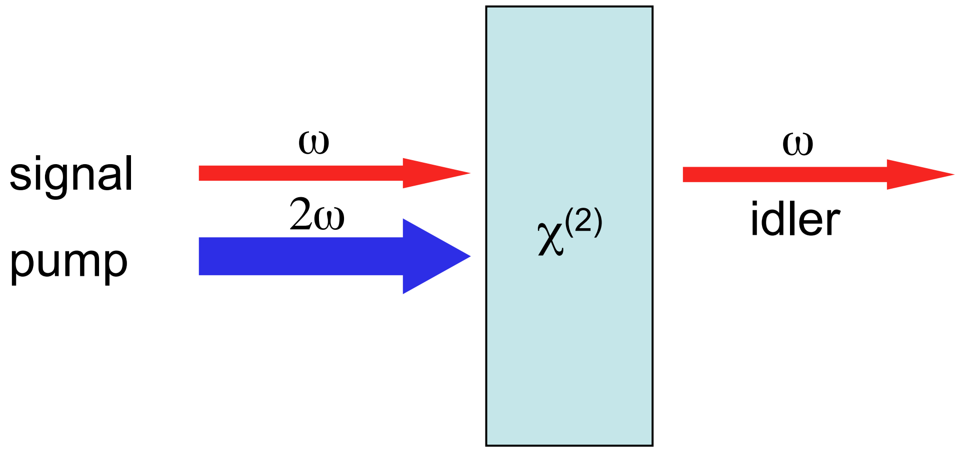
$$\hat{t}_2 = \frac{i}{\sqrt{2}}\left(\hat{t}^{+} - \hat{t}\right) = \frac{i}{\sqrt{2}}\left(\hat{a}^{+} - \hat{a}\right)\left(\cosh r - \sinh r\right) = \hat{X}_2 e^{-r}$$

$$\left\langle \alpha \middle| \hat{S}^{+}(z)\hat{t}_1\hat{S}(z) \middle| \alpha \right\rangle = \frac{\alpha}{2} e^{r}$$

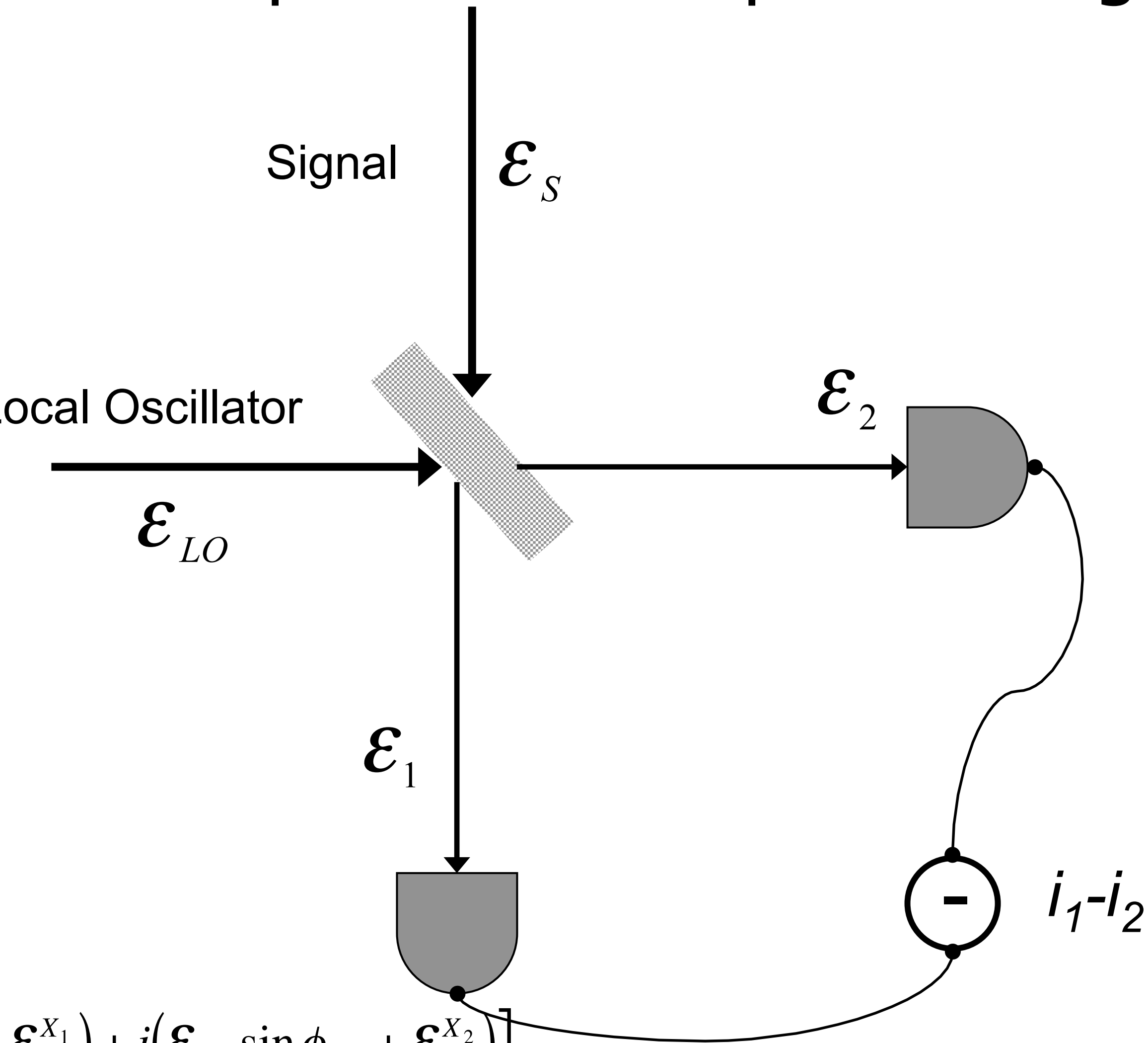
$$\left\langle \alpha \middle| \hat{S}^{+}(z)\hat{t}_2\hat{S}(z) \middle| \alpha \right\rangle = \frac{\alpha}{2} e^{-r}$$

Generation of squeezed light

$$S(z) = e^{\frac{1}{2}\left(za^2 - z^* a^{+2}\right)}$$



Detection of quadrature squeezed light



$$\mathcal{E}_1 = \frac{1}{\sqrt{2}}\left(\mathcal{E}_{LO}e^{i\phi_{LO}} + \mathcal{E}_S\right)$$

$$\mathcal{E}_2 = \frac{1}{\sqrt{2}}\left(\mathcal{E}_{LO}e^{i\phi_{LO}} - \mathcal{E}_S\right)$$

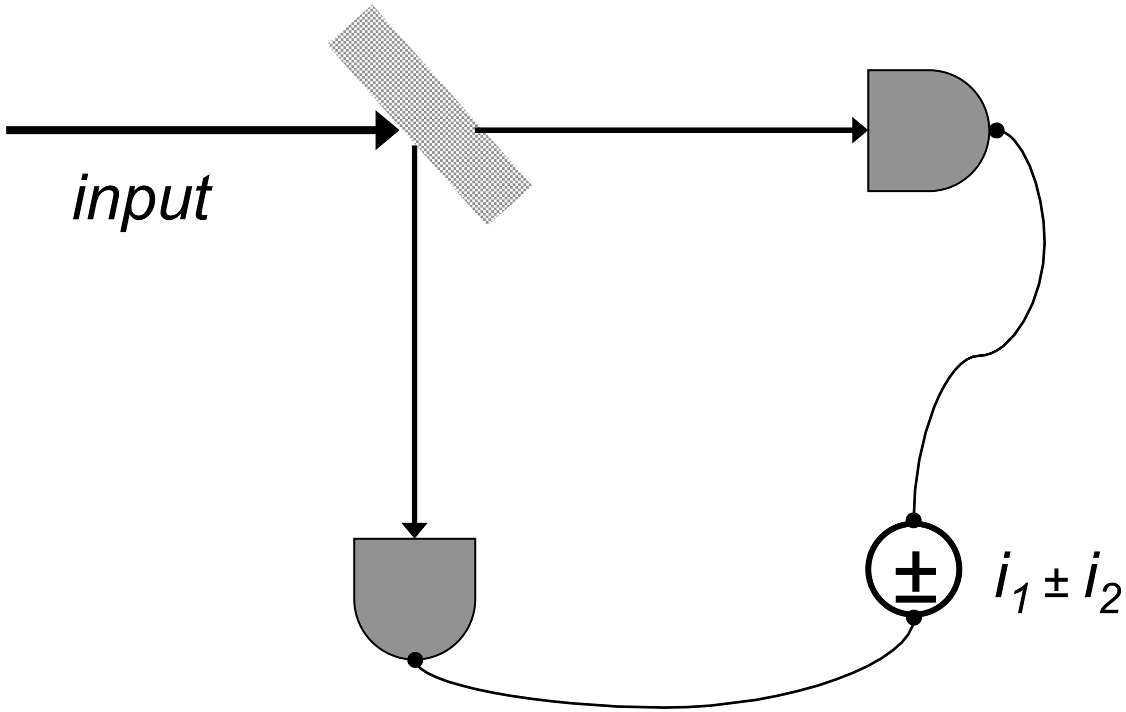
$$\mathcal{E}_S = \mathcal{E}_S^{X_1} + i\mathcal{E}_S^{X_2}$$

$$\mathcal{E}_1 = \frac{1}{\sqrt{2}}\left[\left(\mathcal{E}_{LO}\cos\phi_{LO} + \mathcal{E}_S^{X_1}\right) + i\left(\mathcal{E}_{LO}\sin\phi_{LO} + \mathcal{E}_S^{X_2}\right)\right]$$

$$\mathcal{E}_2 = \frac{1}{\sqrt{2}}\left[\left(\mathcal{E}_{LO}\cos\phi_{LO} - \mathcal{E}_S^{X_1}\right) + i\left(\mathcal{E}_{LO}\sin\phi_{LO} - \mathcal{E}_S^{X_2}\right)\right]$$

$$\left(i_1 - i_2\right) \propto \mathcal{E}_1\mathcal{E}_1^* - \mathcal{E}_2\mathcal{E}_2^* \propto 2\mathcal{E}_{LO}\left(\cos\phi_{LO}\mathcal{E}_S^{X_1} + \sin\phi_{LO}\mathcal{E}_S^{X_2}\right)$$

Detection of number squeezing

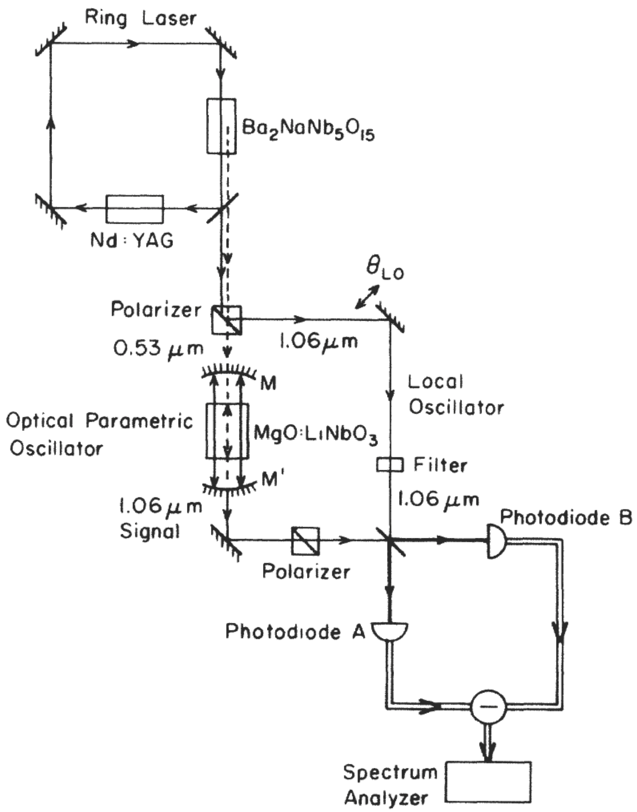


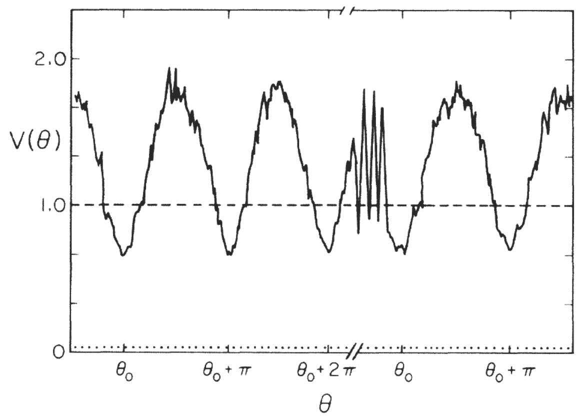
Demonstrations of squeezed light I

VOLUME 57, NUMBER 20 PHYSICAL REVIEW LETTERS 17 NOVEMBER 1986

Generation of Squeezed States by Parametric Down Conversion

Ling-An Wu, H. J. Kimble, J. L. Hall,[(a)] and Huifa Wu
Department of Physics, University of Texas at Austin, Austin, Texas 78712



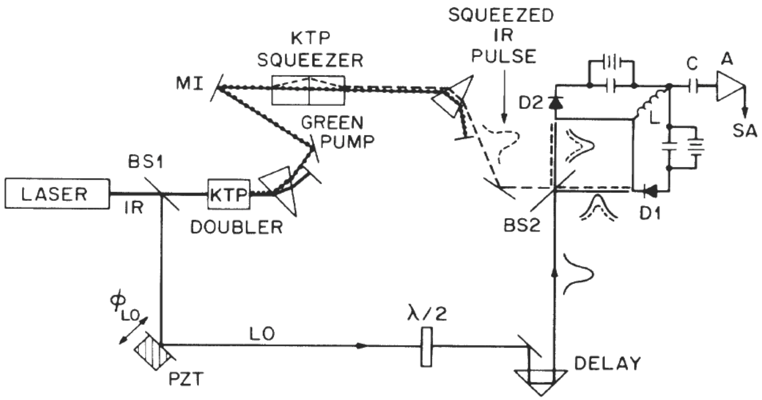


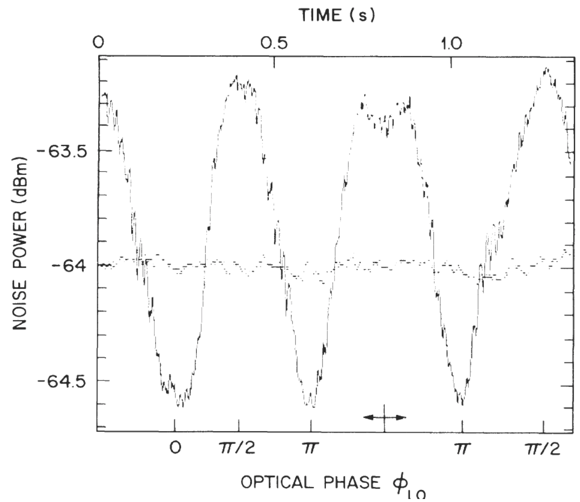
Demonstrations of squeezed light II

VOLUME 59, NUMBER 22 PHYSICAL REVIEW LETTERS 30 NOVEMBER 1987

Pulsed Squeezed Light

R. E. Slusher, P. Grangier, A. LaPorta, B. Yurke, and M. J. Potasek
AT&T Bell Laboratories, Murray Hill, New Jersey 07974
(Received 21 September 1987)





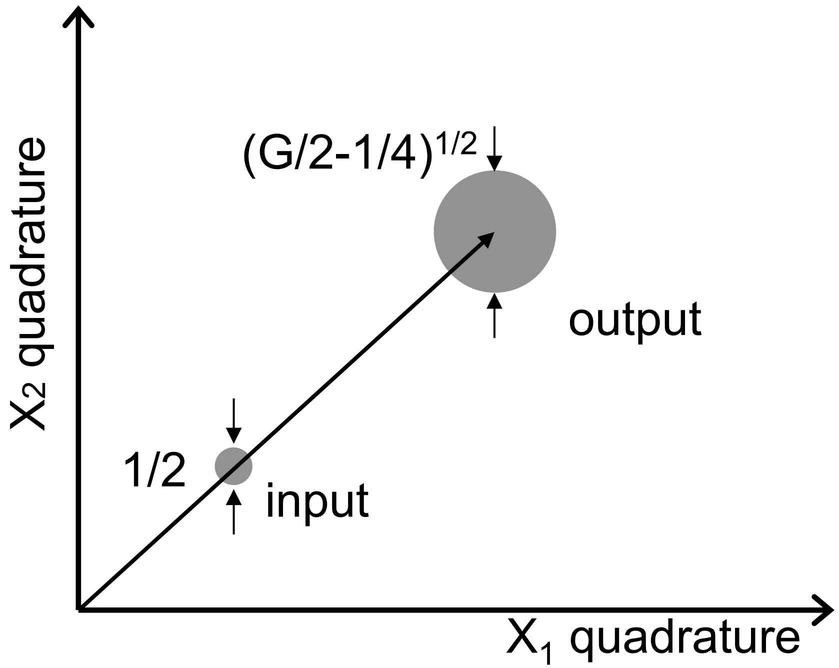
Quantum noise in amplifiers

$$SNR = \frac{(\text{signal amplitude})^2}{(\text{noise amplitude})^2} = \frac{\langle I \rangle^2}{\langle \Delta I \rangle^2}$$

$$SNR = \frac{\bar{n}^2}{(\Delta n)^2}$$

$$G = e^{\gamma L}$$

$$\text{Noise figure} = \frac{SNR_{in}}{SNR_{out}} = 2 - \frac{1}{G}$$



Exercises

1. Use the definition of field quadratures $X_1(t)$ and $X_2(t)$ to verify what are their physical dimensions.

2. A ruby laser operating at 693nm emits pulses of energy 1mJ. Calculate the uncertainty in the phase of the laser light .

3. The proposed Laser Interferometer Space Antenna (LISA) experiment for gravity wave detection will use a standard Michelson interferometer (i.e. no power reciclying or cavity enhancement) with a laser operating at 1064 nm The length if the arms of the interferometer is 5×10^6 km and the power of the beams that form the interference pattern is ~10^{-11} W. Calculate the minimum strain that can be detected.

4. !! Demonstrate that an elastic force $F=-kx(1-\varepsilon \sin 2\omega_0 t)$ modulated at twice the natural frequency ($\varepsilon<<1$) produces on an object of mass m oscillations that grow in time at one quadrature, while are damped at the other quadrature.