Quantum theory of photodetection

Coherent states II

$$\langle\alpha|\alpha\rangle = |c_0|^2 \sum_{m,n=0}^{\infty} \frac{\alpha^{*m}\alpha^n}{\sqrt{m!n!}}\langle m|n\rangle = |c_0|^2 \sum_{n=0}^{\infty} \frac{\left(|\alpha|^2\right)^n}{\sqrt{n!}} = |c_0|^2 e^{|\alpha|^2} = 1$$

$$c_0 = e^{-\frac{|\alpha|^2}{2}}$$

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

Photons, energy and field in coherent states

$$\bar{n} = \langle\alpha|\hat{n}|\alpha\rangle = \langle\alpha|\hat{a}^\dagger\hat{a}|\alpha\rangle = \langle\alpha|\alpha^*\alpha|\alpha\rangle = |\alpha|^2$$

$$E = \langle\alpha|\hat{H}|\alpha\rangle = \hbar\omega\left(\langle\alpha|\hat{a}^\dagger\hat{a}|\alpha\rangle + \frac{1}{2}\right) = \hbar\omega\left(|\alpha|^2 + \frac{1}{2}\right)$$

$$\hat{\mathcal{E}}_x = \sqrt{\frac{\hbar\omega}{\varepsilon_0 V}}\left(\hat{a} + \hat{a}^\dagger\right)\sin kx$$

$$\overline{\mathcal{E}}_x = \langle\alpha|\hat{\mathcal{E}}_x|\alpha\rangle = \sqrt{\frac{\hbar\omega}{\varepsilon_0 V}}\sin kx\langle\alpha|\left(\hat{a} + \hat{a}^\dagger\right)|\alpha\rangle = \sqrt{\frac{\hbar\omega}{\varepsilon_0 V}}\sin kx\left(\alpha + \alpha^*\right)$$

Poisson distribution

$$\bar{n} = |\alpha|^2$$

$$(\Delta n)^2 = \langle\alpha|(\hat{n} - \bar{n})^2|\alpha\rangle = \langle\alpha|\hat{n}^2|\alpha\rangle - 2\bar{n}\langle\alpha|\hat{n}|\alpha\rangle + \bar{n}^2\langle\alpha|\alpha\rangle$$

$$= \langle\alpha|\hat{a}^\dagger\hat{a}\hat{a}^\dagger\hat{a}|\alpha\rangle - 2\bar{n}^2 + \bar{n}^2 = \langle\alpha|\hat{a}^\dagger\hat{a}|\alpha\rangle + \langle\alpha|\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}|\alpha\rangle - \bar{n}^2$$

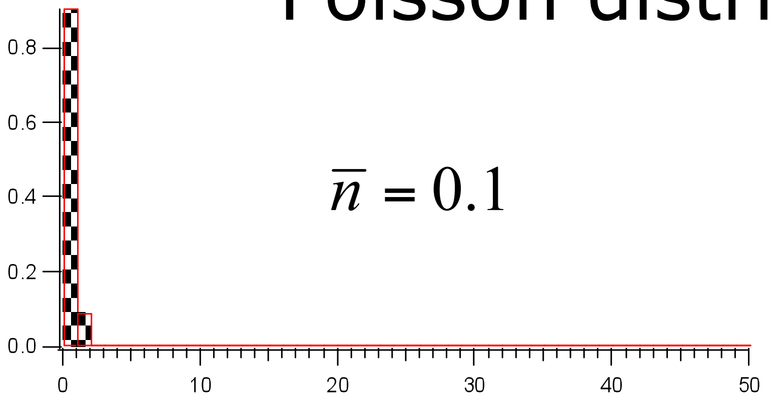
$$= \bar{n} + \bar{n}^2 - \bar{n}^2 = \bar{n}$$

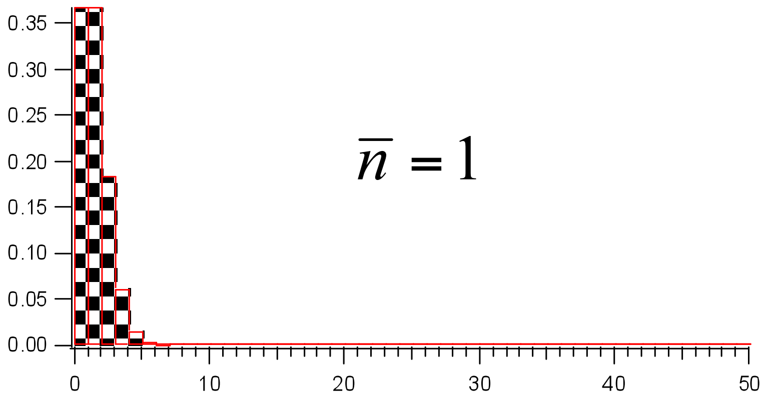
$$\Delta n = \sqrt{\bar{n}}$$

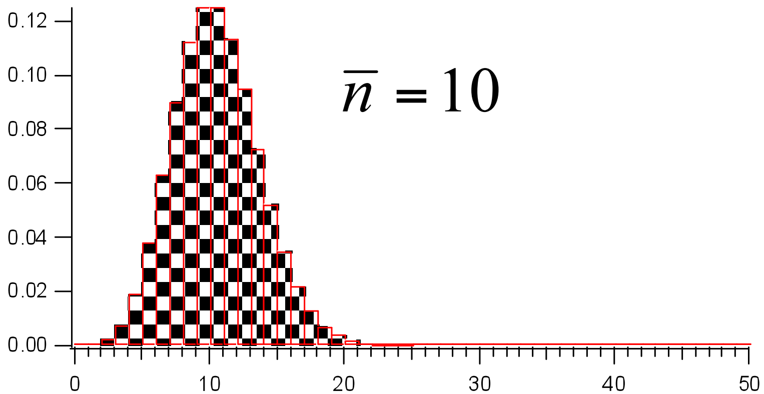
$$P(n) = |\langle n|\alpha\rangle|^2 = \left| e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \langle n|n\rangle \right|^2 = e^{-|\alpha|^2} \frac{\left(|\alpha|^2\right)^n}{n!} = \frac{\bar{n}^n}{n!} e^{-\bar{n}}$$

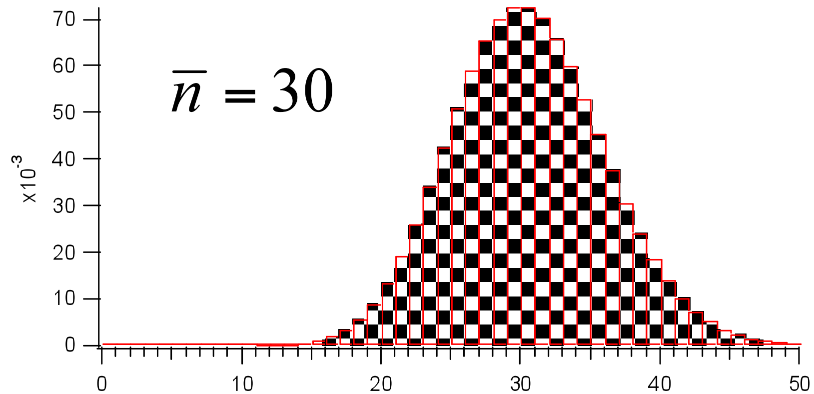
Poisson distribution - examples

$$P(n) = \frac{\overline{n}^{\,n}}{n!} e^{-\overline{n}}$$



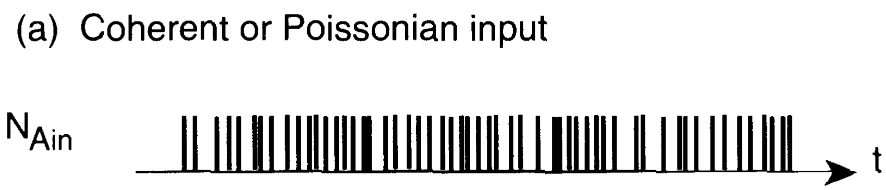


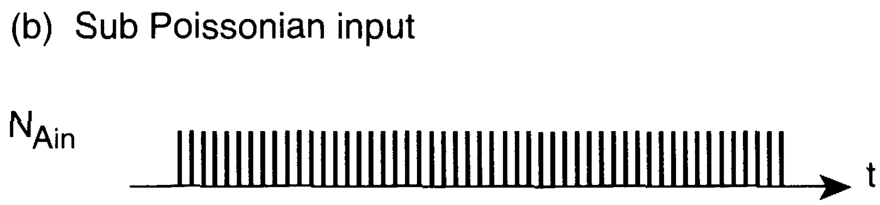


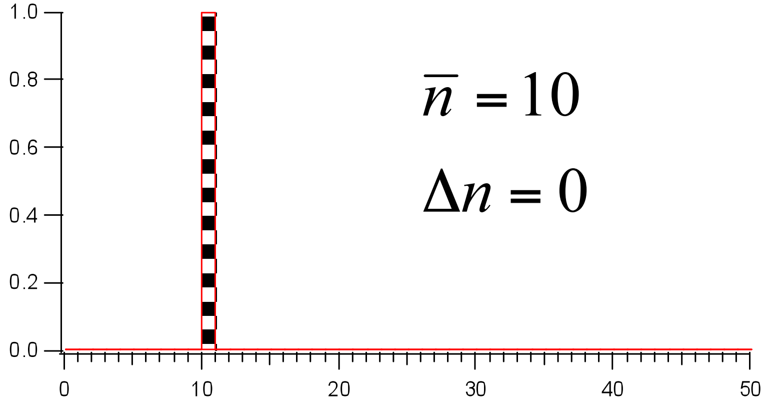


Sub-Poissonian light

$$\Delta n < \sqrt{\bar{n}}$$



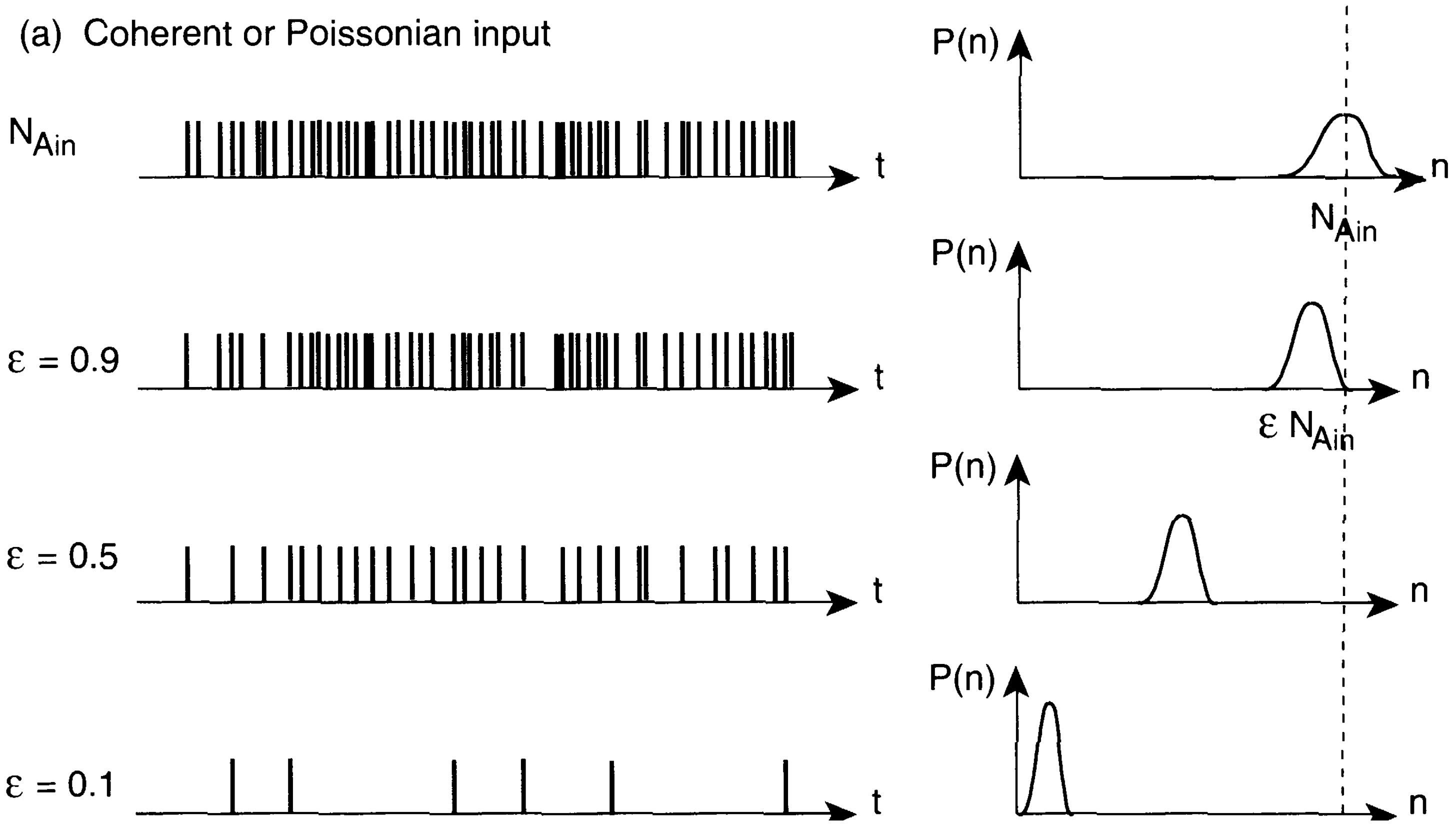




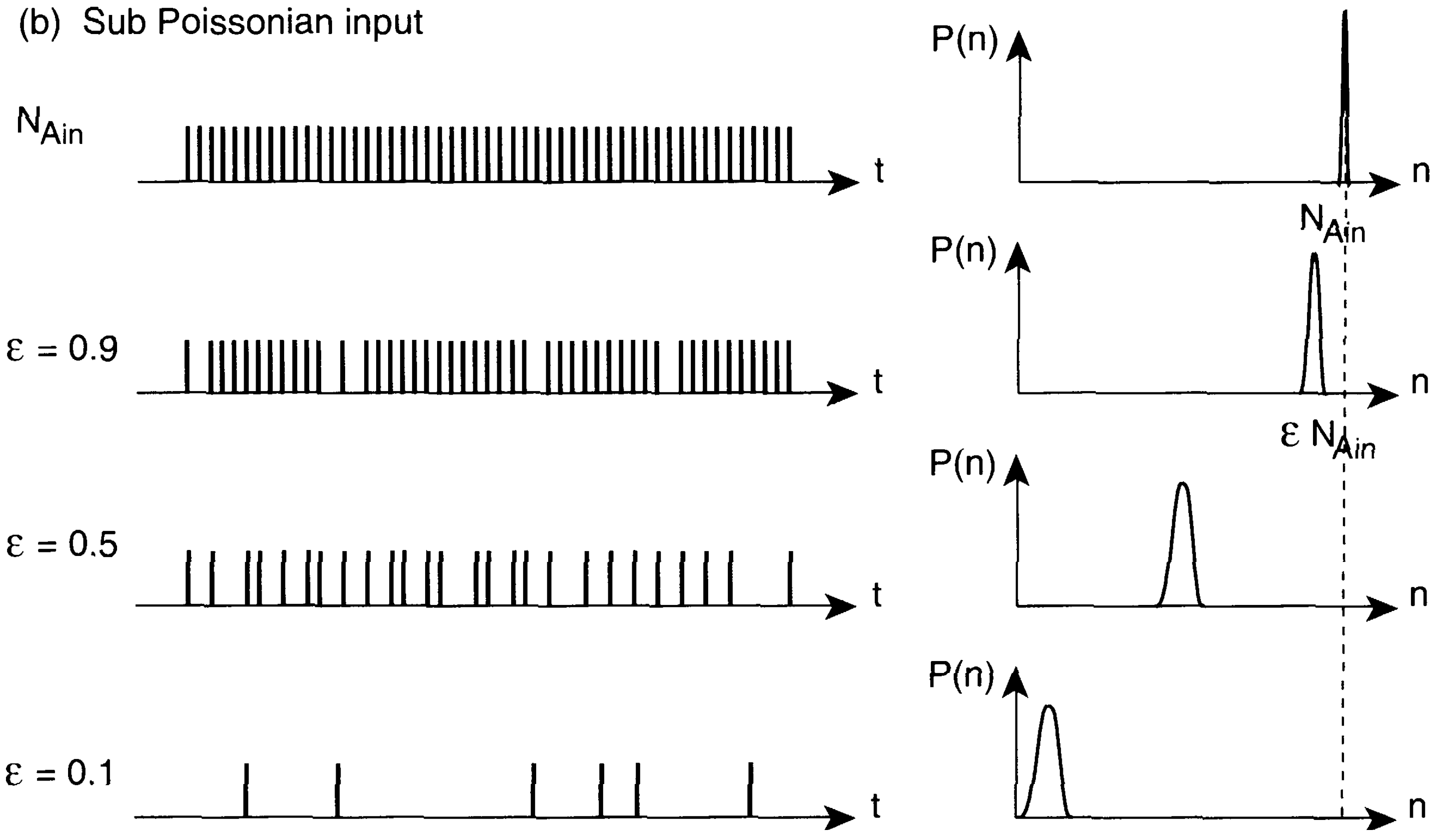
$$\bar{n} = 10$$

$$\Delta n = 0$$

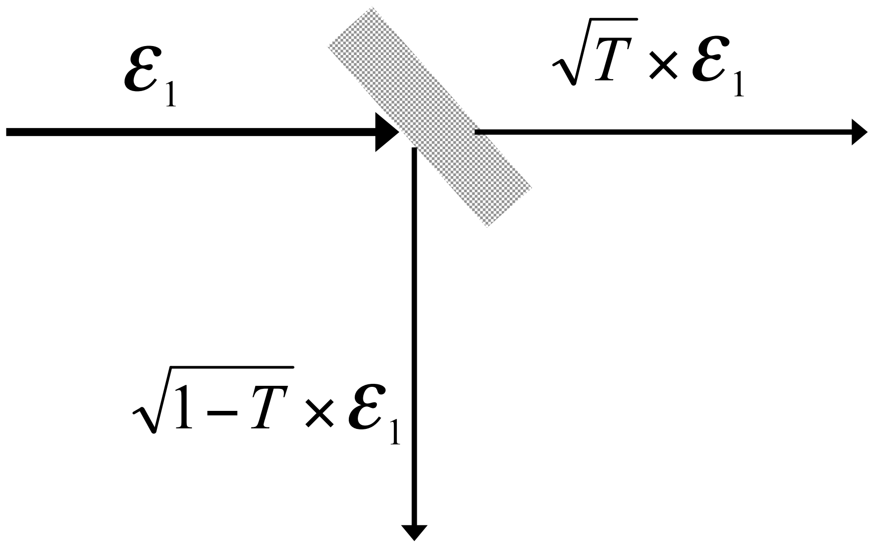
Photon statistics and losses I



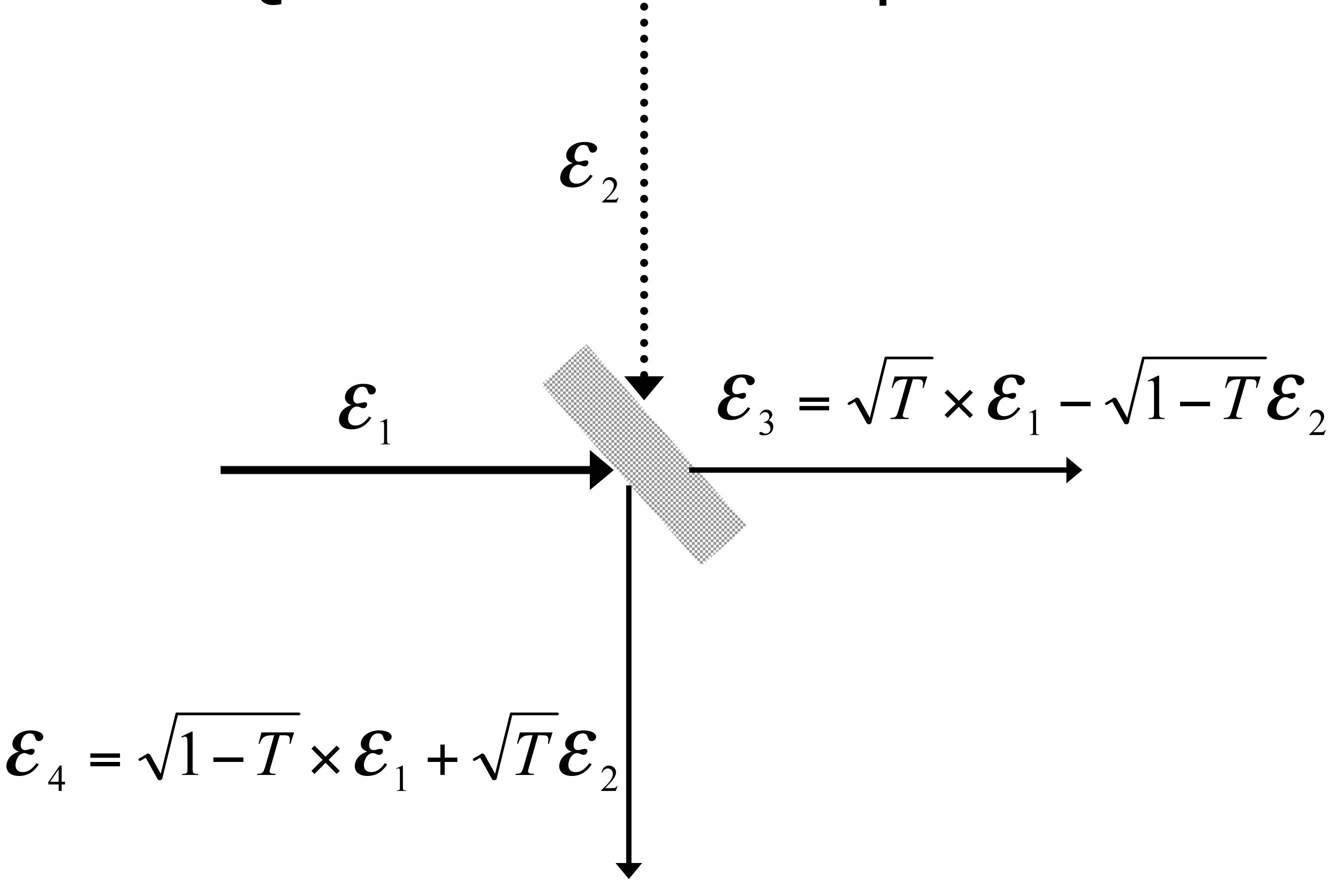
Photon statistics and losses II



Losses as a beam splitter



Quantum beam splitter

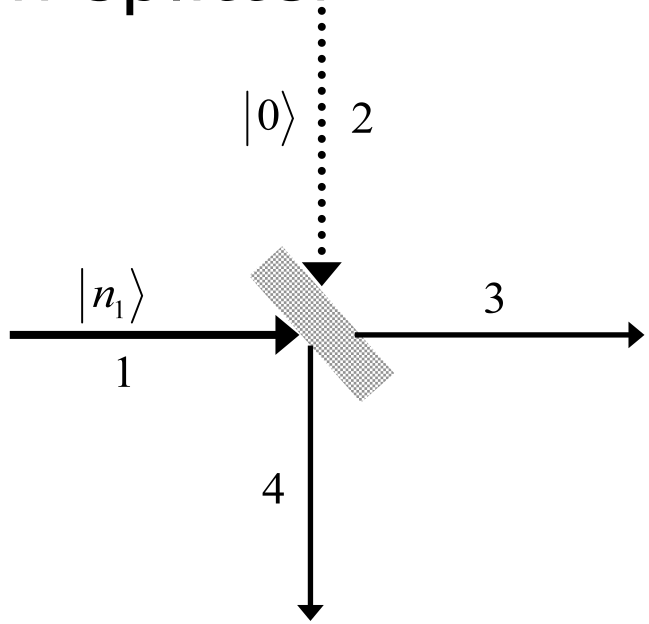


$$\hat{a}_1, \hat{a}_2$$

$$\hat{a}_3 = \sqrt{T}\hat{a}_1 - \sqrt{1-T}\hat{a}_2$$

$$\hat{a}_4 = \sqrt{1-T}\hat{a}_1 + \sqrt{T}\hat{a}_2$$

Losses as a quantum beam splitter



$$|\psi\rangle = |n_1, 0_2\rangle$$

$$\bar{n}_3 = \langle \hat{a}_3^\dagger \hat{a}_3 \rangle = \langle n_1, 0_2 | \hat{a}_3^\dagger \hat{a}_3 | n_1, 0_2 \rangle =$$

$$= \langle n_1, 0_2 | \left(\sqrt{T} \hat{a}_1^\dagger - \sqrt{1-T} \hat{a}_2^\dagger \right) \left(\sqrt{T} \hat{a}_1 - \sqrt{1-T} \hat{a}_2 \right) | n_1, 0_2 \rangle =$$

$$= \langle n_1, 0_2 | \left(\sqrt{T} \hat{a}_1^\dagger \right) \left(\sqrt{T} \hat{a}_1 \right) | n_1, 0_2 \rangle = T \langle n_1, 0_2 | \hat{a}_1^\dagger \hat{a}_1 | n_1, 0_2 \rangle = T n_1$$

$$(\Delta n_3)^2 = \langle \hat{n}_3^2 \rangle - \bar{n}_3^2 = \langle \hat{a}_3^\dagger \hat{a}_3 \hat{a}_3^\dagger \hat{a}_3 \rangle - \bar{n}_3^2 = \langle \hat{a}_3^\dagger \hat{a}_3^\dagger \hat{a}_3 \hat{a}_3 \rangle + \langle \hat{a}_3^\dagger \hat{a}_3 \rangle - \bar{n}_3^2 = \langle \hat{a}_3^\dagger \hat{a}_3^\dagger \hat{a}_3 \hat{a}_3 \rangle + \bar{n}_3 - \bar{n}_3^2$$

$$\langle \hat{a}_3^\dagger \hat{a}_3^\dagger \hat{a}_3 \hat{a}_3 \rangle = \langle n_1, 0_2 | \left(\sqrt{T} \hat{a}_1^\dagger - \sqrt{1-T} \hat{a}_2^\dagger \right) \left(\sqrt{T} \hat{a}_1^\dagger - \sqrt{1-T} \hat{a}_2^\dagger \right) \left(\sqrt{T} \hat{a}_1 - \sqrt{1-T} \hat{a}_2 \right) \left(\sqrt{T} \hat{a}_1 - \sqrt{1-T} \hat{a}_2 \right) | n_1, 0_2 \rangle =$$

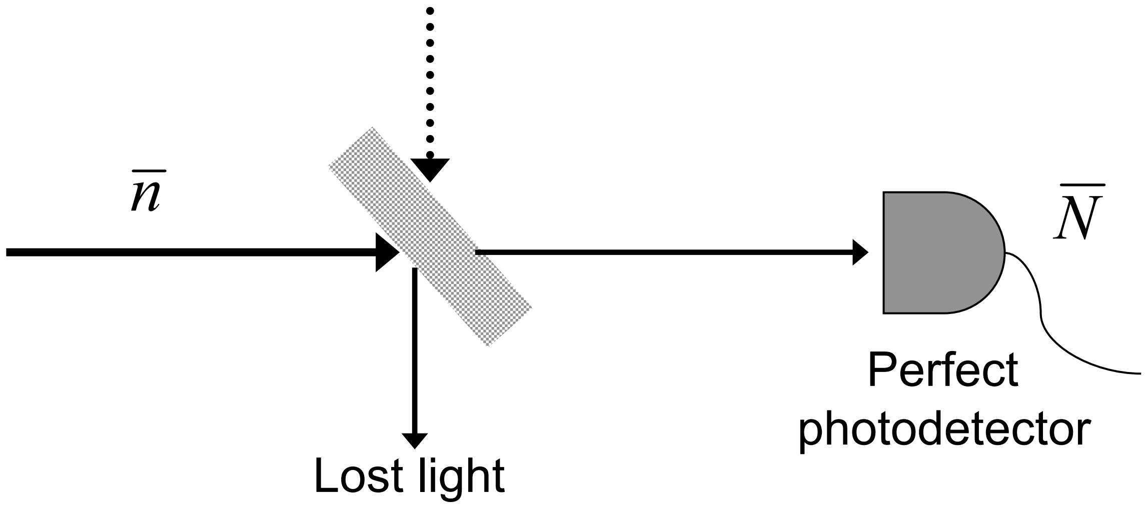
$$= T^2 \langle n_1, 0_2 | \hat{a}_1^\dagger \hat{a}_1^\dagger \hat{a}_1 \hat{a}_1 | n_1, 0_2 \rangle = T^2 \langle n_1, 0_2 | \left(\hat{a}_1^\dagger \hat{a}_1 \hat{a}_1^\dagger \hat{a}_1 - \hat{a}_1^\dagger \hat{a}_1 \right) | n_1, 0_2 \rangle =$$

$$= T^2 \left(n_1^2 - n_1 \right)$$

$$(\Delta n_3)^2 = T^2 \left(n_1^2 - n_1 \right) + T n_1 - T^2 n_1^2 = T n_1 (1 - T)$$

$$(\Delta n_3)^2 = n_3 (1 - T)$$

Quantum theory of photodetection



$$\eta = \frac{\overline{N}}{\overline{n}}$$ Quantum efficiency

$$(\Delta N)^2 = \eta^2 (\Delta n)^2 + \eta(1-\eta)\overline{n}$$

Circuit noise: thermal noise

Thermal noise (or *Johnson noise*) arises from random motion of mobile carriers in resistive electrical materials at finite temperatures

Random electric current i(t) in the absence of an external power source

$$\langle i(t) \rangle = 0$$

Power spectral density of thermal noise

$$S_{\mathrm{th}}(f) = \frac{4}{R} \frac{hf}{\exp\left(\dfrac{hf}{k_{\mathrm{B}}T}\right) - 1}$$

$$S_{\mathrm{th}}(f) \approx \frac{4k_{\mathrm{B}}T}{R}$$

for $f \ll k_{\mathrm{B}}T / h = 6.25\,\mathrm{THz}$ (300K)

Noise variance

$$\sigma_i^2 = \int_0^B S_{\mathrm{th}}(f)df \approx \frac{4k_{\mathrm{B}}T}{R} B$$

Observation of sub-Poissonian statistics

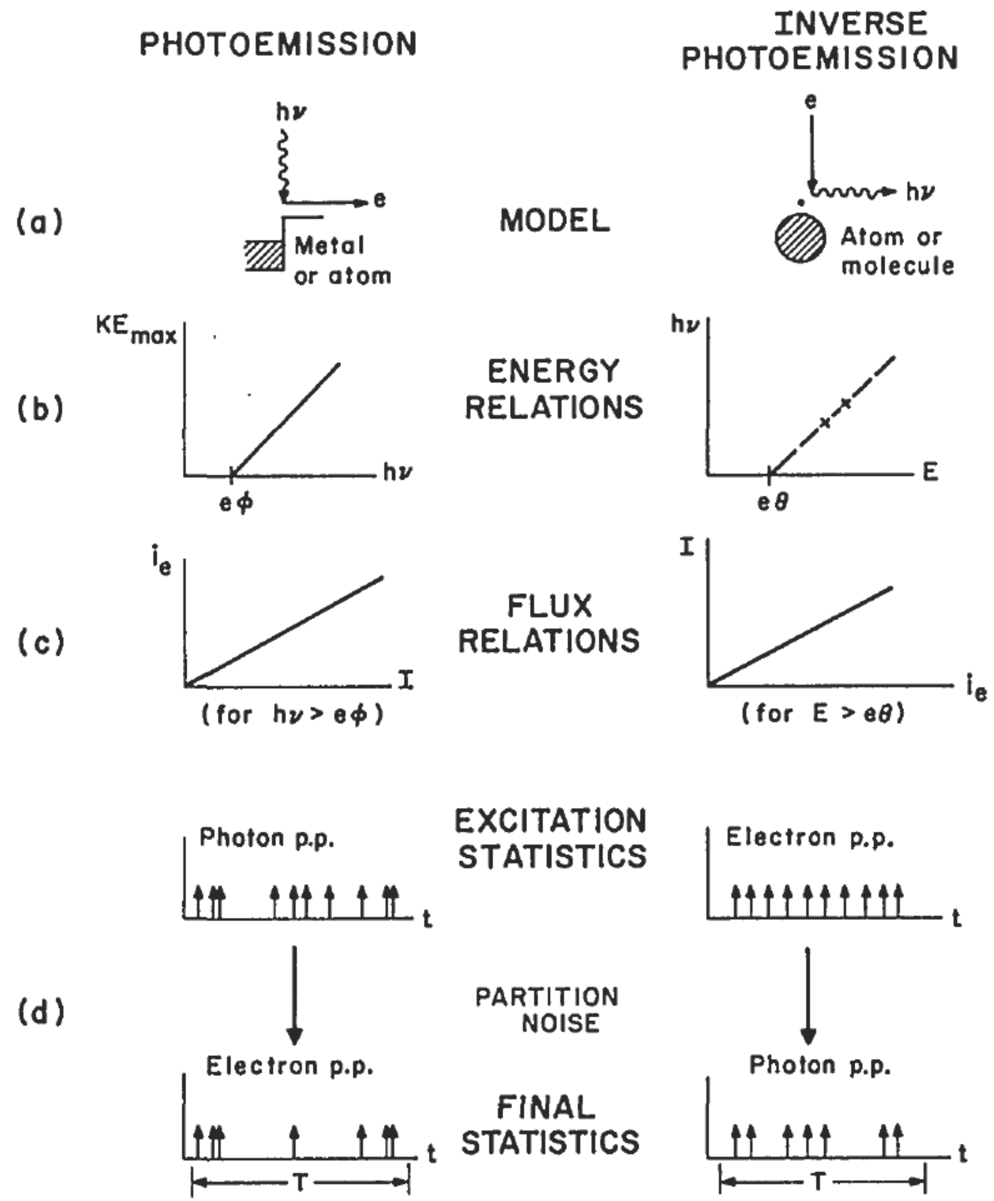


Fig. 1. Schematic representation of photoemission and inverse photoemission (Franck–Hertz effect). Electrons and photons behave like classical particles in photon counting, provided that $M_s \gg 1$, $T \gg \tau_p, \tau_e$.

Observation of sub-Poissonian statistics II

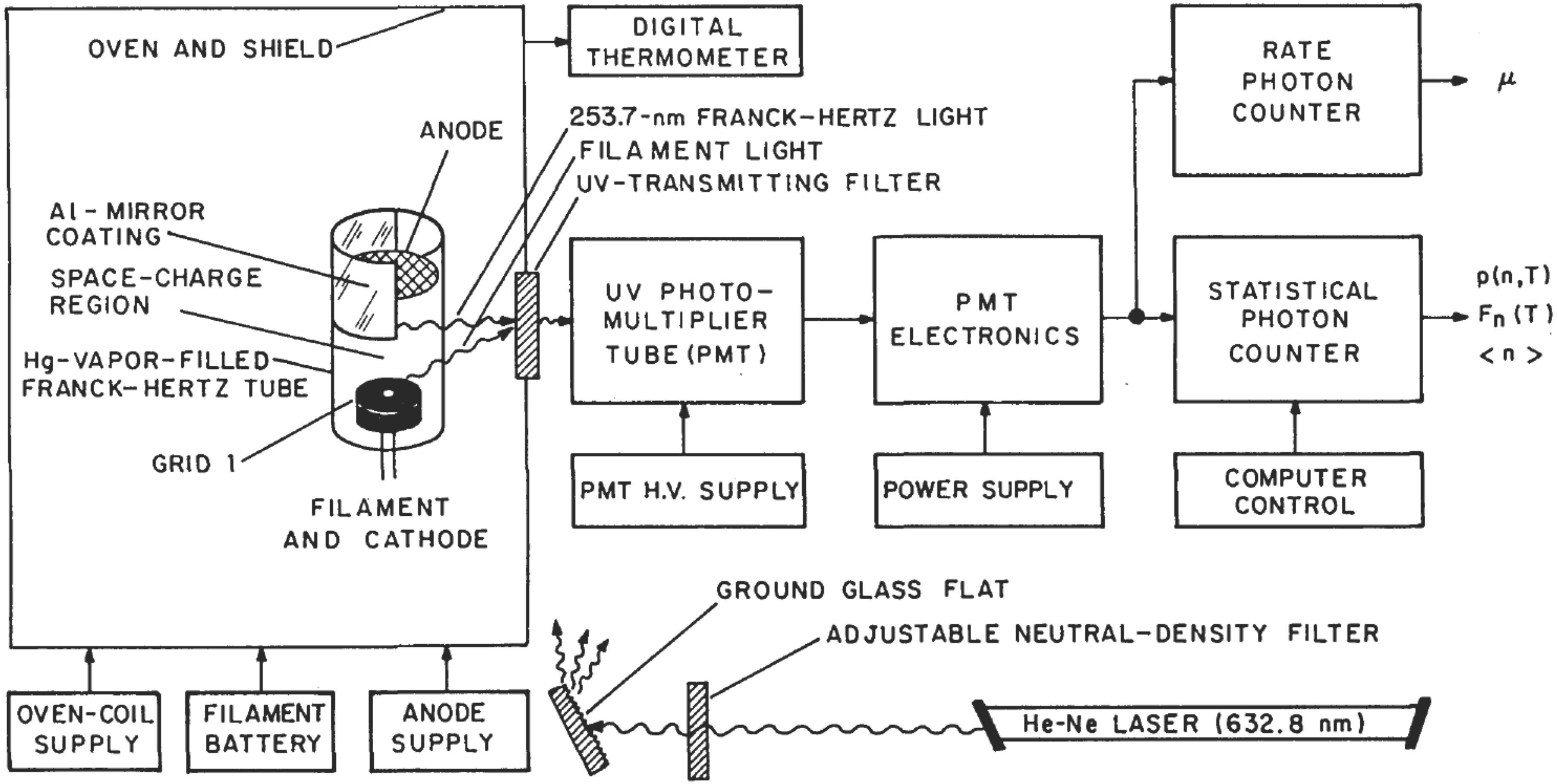
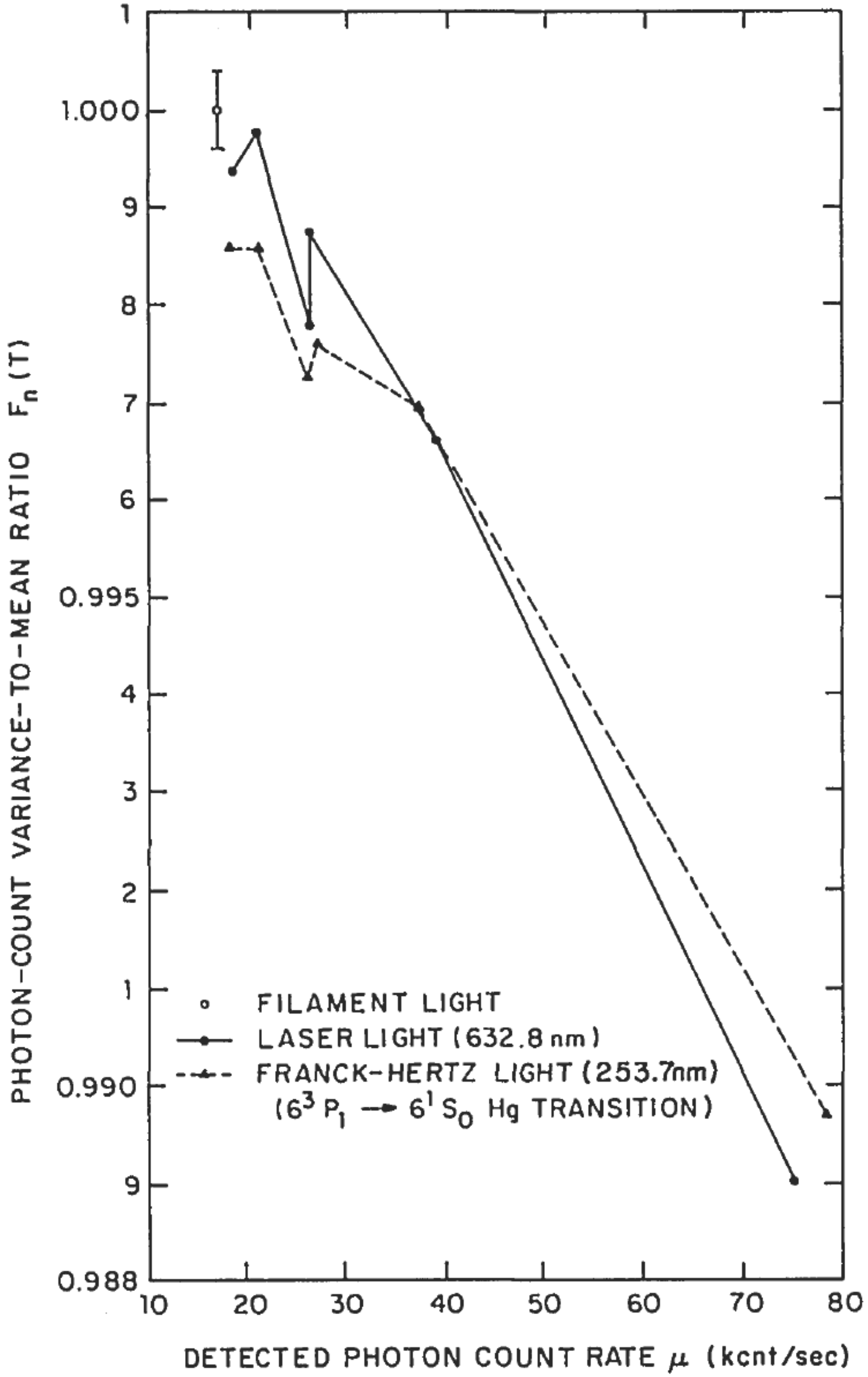


Fig. 2. Block diagram of the experimental arrangement.

Observation of sub-Poissonian statistics III

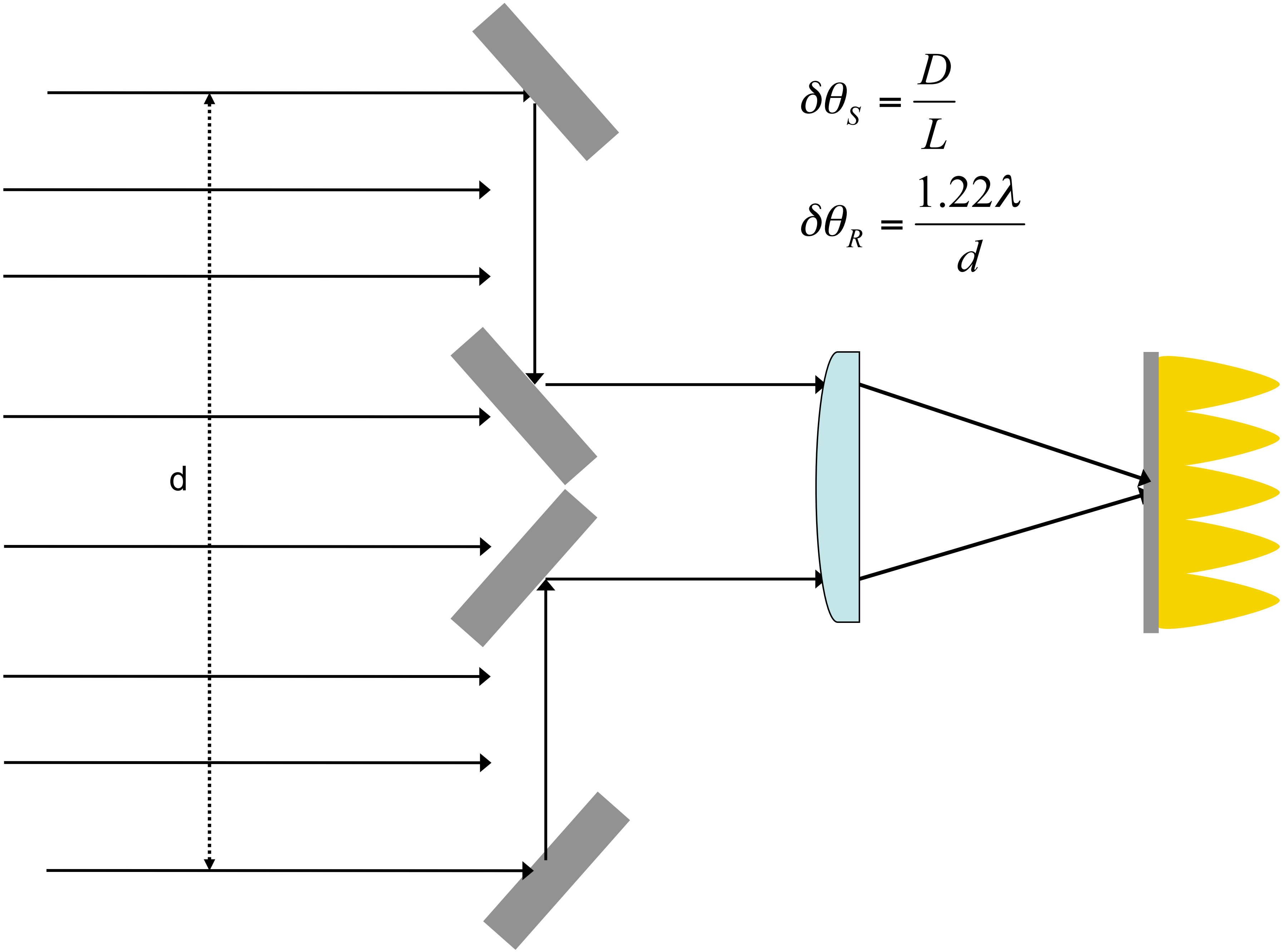


Coherence and Correlation

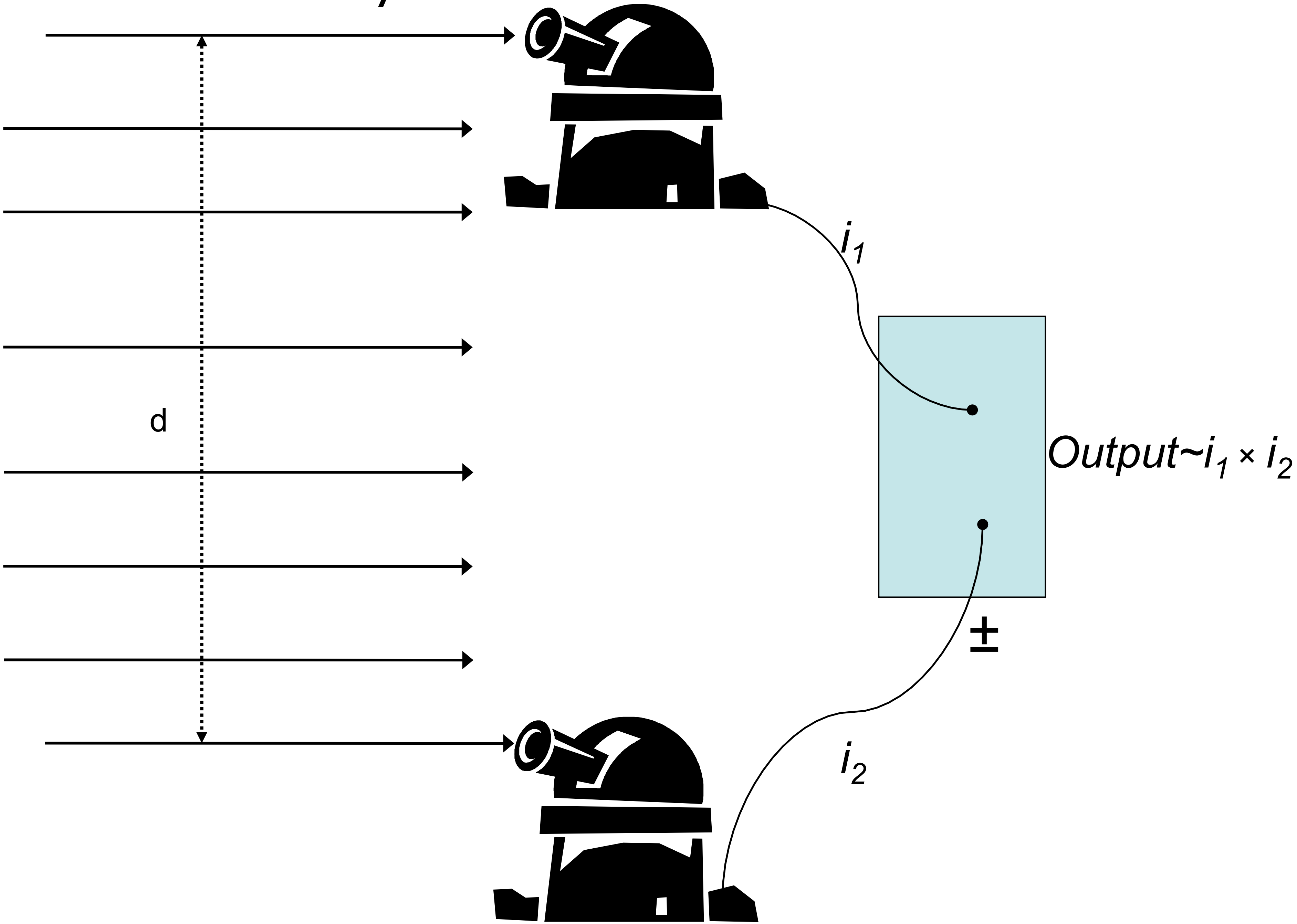
Michelson interferometer

$$\delta\theta_S = \frac{D}{L}$$

$$\delta\theta_R = \frac{1.22\lambda}{d}$$



Hanbury-Brown Twiss interferometer



g1 and coherence

$$g^{(1)}(\tau) = \frac{\left\langle \mathcal{E}^*(t)\mathcal{E}(t+\tau) \right\rangle}{\left\langle \left|\mathcal{E}(t)\right|^2 \right\rangle}$$

$$\left\langle \mathcal{E}^*(t)\mathcal{E}(t+\tau) \right\rangle = \frac{1}{T}\int_T \mathcal{E}^*(t)\mathcal{E}(t+\tau)\,dt$$

$$g^{(1)}(0) = 1$$

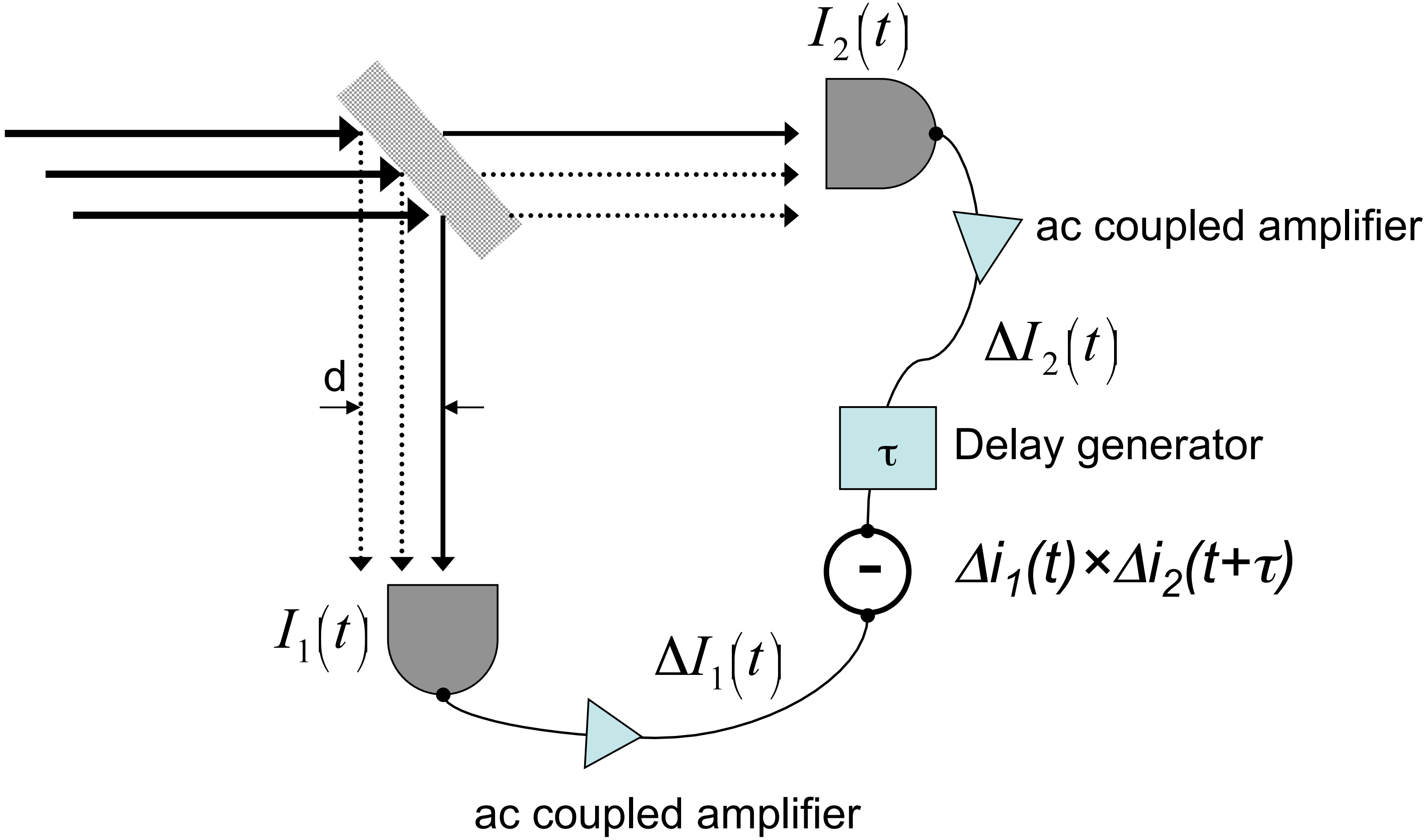
$$\mathcal{E}(t) = \mathcal{E}_0(t)e^{-i\omega_0 t}e^{i\phi} \Rightarrow g^{(1)}(\tau) = e^{-i\omega_0 t}\left\langle e^{i[\phi(t-\tau)-\phi(t)]} \right\rangle$$

$$g^{(1)}(\tau) = e^{-i\omega_0 t}e^{-\frac{|\tau|}{t_C}}$$

$$t_C = \frac{1}{\Delta\omega}$$

$$visibility = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \left|g^{(1)}(\tau)\right|$$

HBT experiment deconstructed



g2 and correlation

$$g^{(2)}(\tau) = \frac{\left\langle \mathcal{E}^*(t)\mathcal{E}^*(t+\tau)\mathcal{E}(t+\tau)\mathcal{E}(t) \right\rangle}{\left\langle \left|\mathcal{E}(t)\right|^2 \right\rangle \left\langle \left|\mathcal{E}(t+\tau)\right|^2 \right\rangle} = \frac{\left\langle I(t)I(t+\tau) \right\rangle}{\left\langle I(t) \right\rangle \left\langle I(t+\tau) \right\rangle}$$

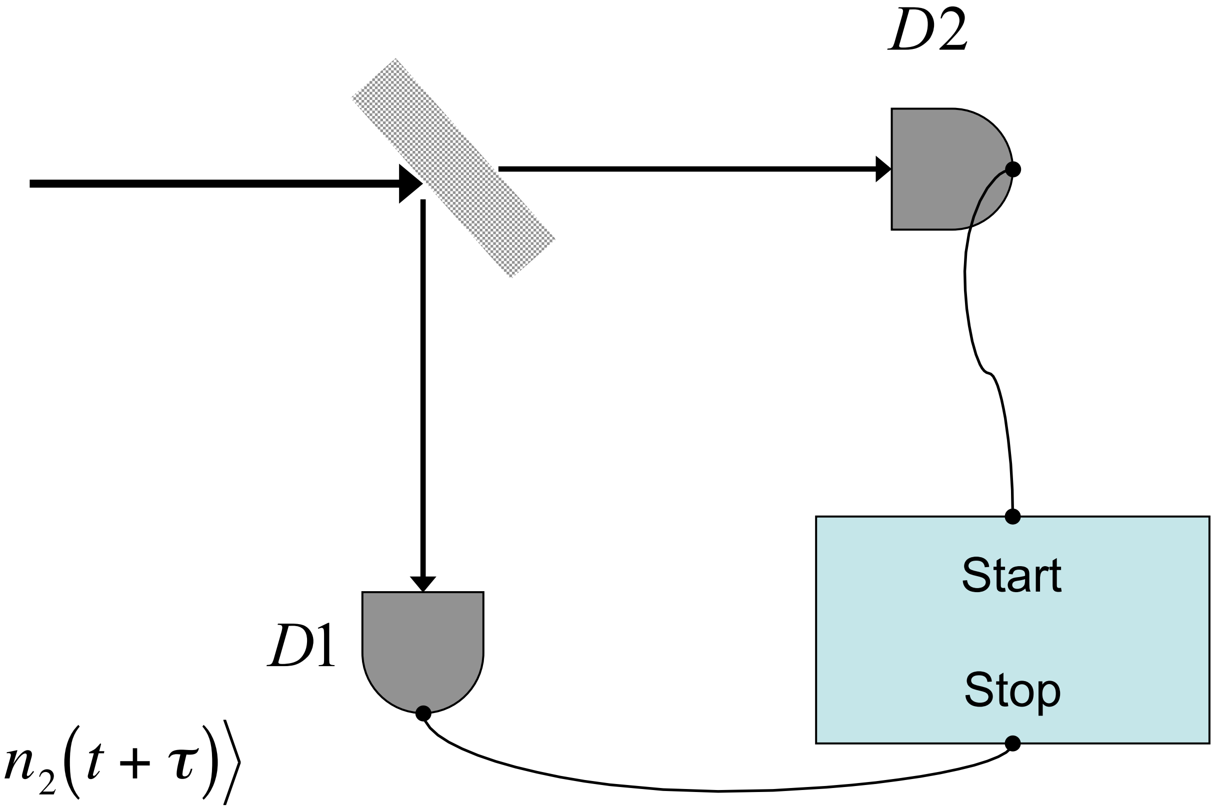
$$g^{(2)}(0) = \frac{\left\langle I(t)I(t+\tau) \right\rangle}{\left\langle I(t) \right\rangle^2} = \frac{\left\langle I(t) \right\rangle^2}{\left\langle I(t) \right\rangle^2} = 1 \quad \text{for perfectly coherent light}$$

$$g^{(2)}(0) = \frac{\left\langle I(t)^2 \right\rangle}{\left\langle I(t) \right\rangle^2} \geq 1 \;\text{ and }\; g^{(2)}(0) \geq g^{(2)}(\tau) \quad \text{for any classical light}$$

$$g^{(2)}(\tau) = 1 \qquad \text{for perfectly coherent light}$$

$$g^{(2)}(\tau) = 1 + e^{-\frac{|\tau|}{t_0}} \qquad \text{for chaotic light}$$

HBT experiment with photons



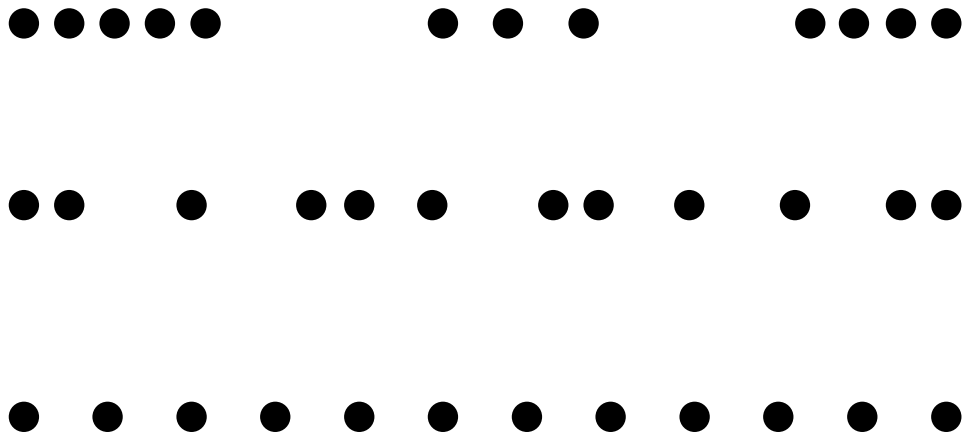
$$g^{(2)}(\tau) = \frac{\langle n_1(t) n_2(t+\tau) \rangle}{\langle n_1(t) \rangle \langle n_2(t+\tau) \rangle}$$

Photon bunching and antibunching

bunching $g^{(2)}(0) > 1$

coherent $g^{(2)}(0) = 1$

antibunching $g^{(2)}(0) < 1$



Quantum theory of HBT

$$\hat{a}_1, \hat{a}_2$$

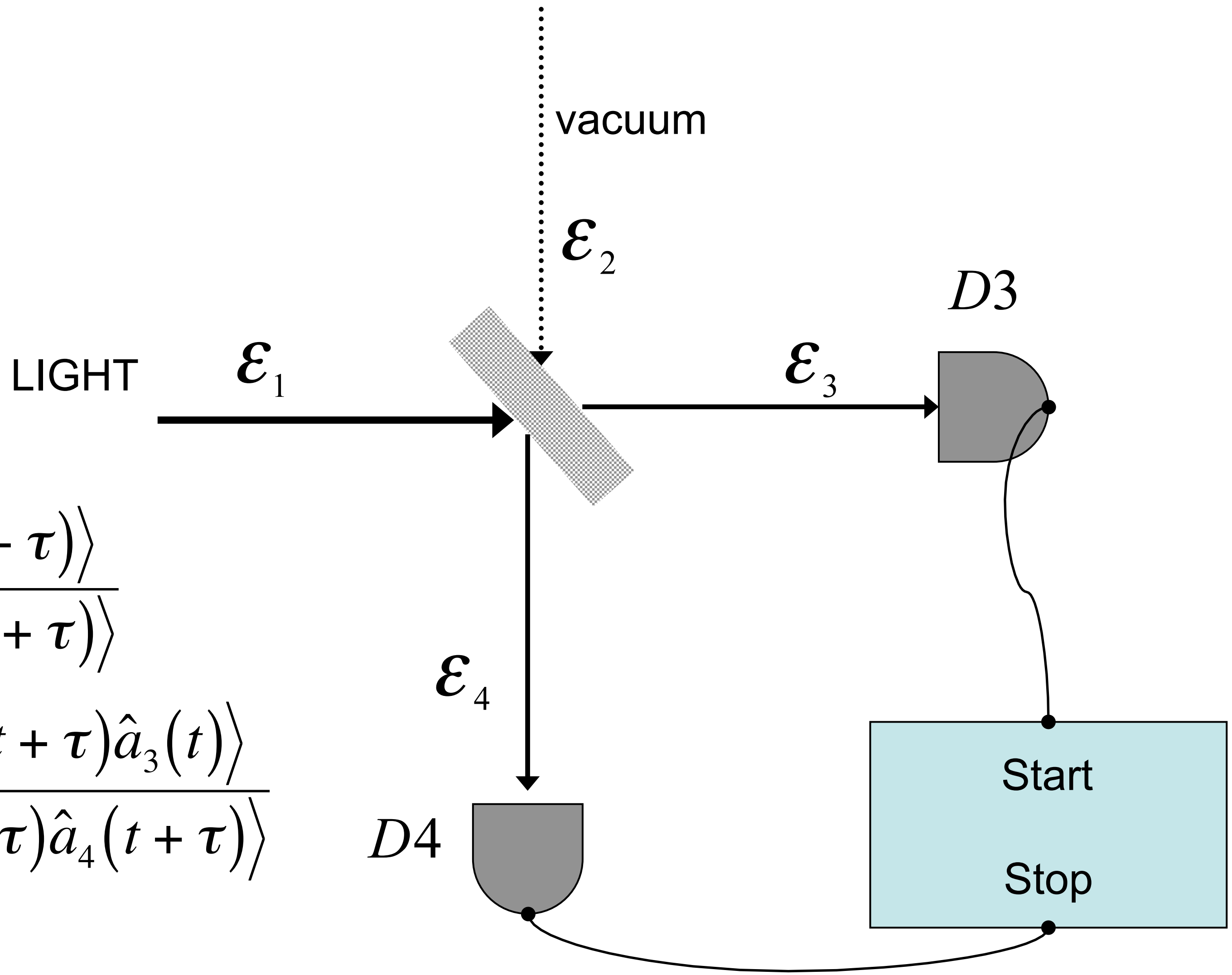
$$\hat{a}_3 = \frac{1}{\sqrt{2}}\left(\hat{a}_1 - \hat{a}_2\right)$$

$$\hat{a}_4 = \frac{1}{\sqrt{2}}\left(\hat{a}_1 + \hat{a}_2\right)$$

$$g^{(2)}(\tau) = \frac{\left\langle n_3(t) n_4(t+\tau)\right\rangle}{\left\langle n_3(t)\right\rangle\left\langle n_4(t+\tau)\right\rangle}$$

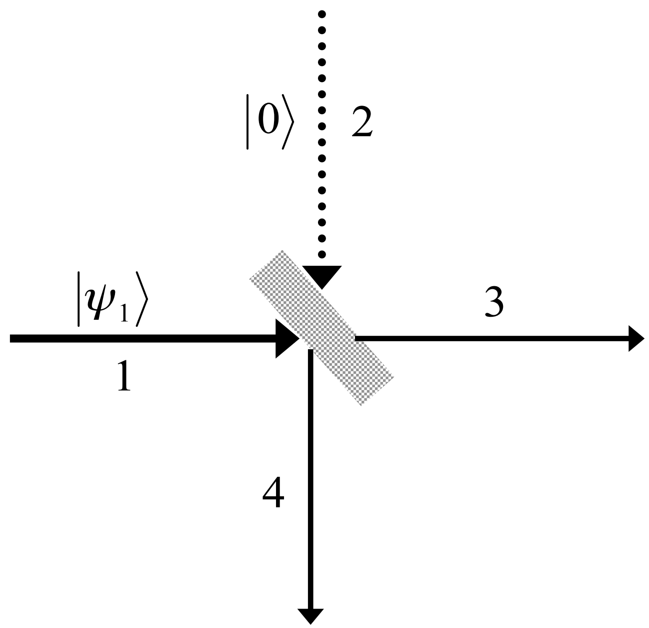
$$= \frac{\left\langle \hat{a}_3^\dagger(t)\hat{a}_4^\dagger(t+\tau)\hat{a}_4(t+\tau)\hat{a}_3(t)\right\rangle}{\left\langle \hat{a}_3^\dagger(t)\hat{a}_3(t)\right\rangle\left\langle \hat{a}_4^+(t+\tau)\hat{a}_4(t+\tau)\right\rangle}$$

$$g^{(2)}(0) = \frac{\left\langle \hat{a}_3^\dagger\hat{a}_4^\dagger\hat{a}_4\hat{a}_3\right\rangle}{\left\langle \hat{a}_3^\dagger\hat{a}_3\right\rangle\left\langle \hat{a}_4^+\hat{a}_4\right\rangle}$$



Quantum theory of HBT II

$$|\psi\rangle = |\psi_1, 0_2\rangle \quad ; \qquad g^{(2)}(0) = \frac{\left\langle \hat{a}_3^\dagger \hat{a}_4^\dagger \hat{a}_4 \hat{a}_3 \right\rangle}{\left\langle \hat{a}_3^\dagger \hat{a}_3 \right\rangle \left\langle \hat{a}_4^+ \hat{a}_4 \right\rangle}$$



$$\bar{n}_3 = \left\langle \hat{a}_3^\dagger \hat{a}_3 \right\rangle = \left\langle \psi_1, 0_2 \right| \hat{a}_3^\dagger \hat{a}_3 \left| \psi_1, 0_2 \right\rangle =$$

$$= \frac{1}{2} \left\langle \psi_1, 0_2 \right| \left(\hat{a}_1^\dagger - \hat{a}_2^\dagger \right) \left(\hat{a}_1 - \hat{a}_2 \right) \left| \psi_1, 0_2 \right\rangle =$$

$$= \frac{1}{2} \left\langle \psi_1, 0_2 \right| \hat{a}_1^\dagger \hat{a}_1 \left| \psi_1, 0_2 \right\rangle = \frac{1}{2} \left\langle \psi_1 \right| \hat{a}_1^\dagger \hat{a}_1 \left| \psi_1 \right\rangle = \frac{\bar{n}_1}{2}$$

$$\bar{n}_4 = \left\langle \hat{a}_4^\dagger \hat{a}_4 \right\rangle = \frac{\bar{n}_1}{2}$$

$$\left\langle \hat{a}_3^\dagger \hat{a}_4^\dagger \hat{a}_4 \hat{a}_3 \right\rangle = \frac{1}{2} \left\langle n_1, 0_2 \right| \left(\hat{a}_1^\dagger - \hat{a}_2^\dagger \right) \left(\hat{a}_1^\dagger - \hat{a}_2^\dagger \right) \left(\hat{a}_1 - \hat{a}_2 \right) \left(\hat{a}_1 - \hat{a}_2 \right) \left| n_1, 0_2 \right\rangle =$$

$$= \frac{1}{4} \left\langle n_1, 0_2 \right| \hat{a}_1^\dagger \hat{a}_1^\dagger \hat{a}_1 \hat{a}_1 \left| n_1, 0_2 \right\rangle = \frac{1}{4} \left\langle n_1, 0_2 \right| \left(\hat{a}_1^\dagger \hat{a}_1 \hat{a}_1^\dagger \hat{a}_1 - \hat{a}_1^\dagger \hat{a}_1 \right) \left| n_1, 0_2 \right\rangle =$$

$$= \frac{1}{4} \left\langle n_1, 0_2 \right| \hat{n}_1 \left(\hat{n}_1 - 1 \right) \left| n_1, 0_2 \right\rangle = \frac{1}{4} \left\langle \psi_1 \right| \hat{n}_1 \left(\hat{n}_1 - 1 \right) \left| \psi_1 \right\rangle$$

$$g^{(2)}(0) = \frac{\left\langle \psi_1 \right| \hat{n}_1 \left(\hat{n}_1 - 1 \right) \left| \psi_1 \right\rangle}{\bar{n}_1^2}$$

Experiments on antibunching

Photon Antibunching in Resonance Fluorescence

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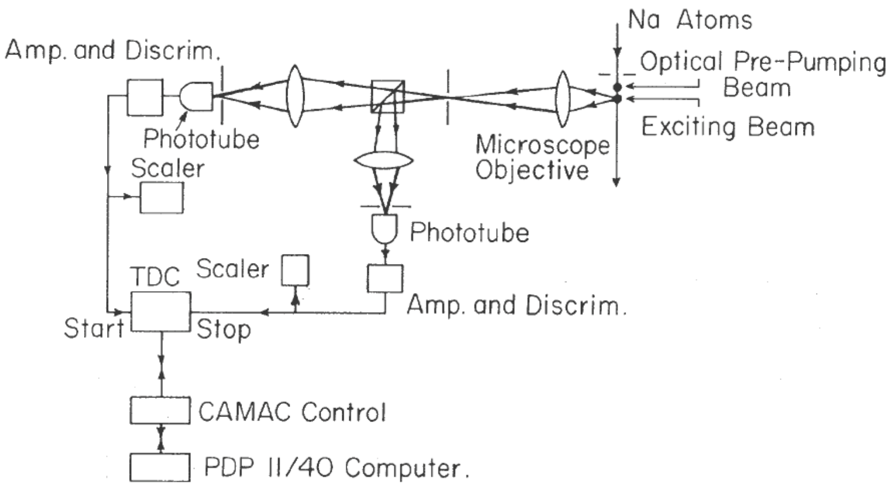
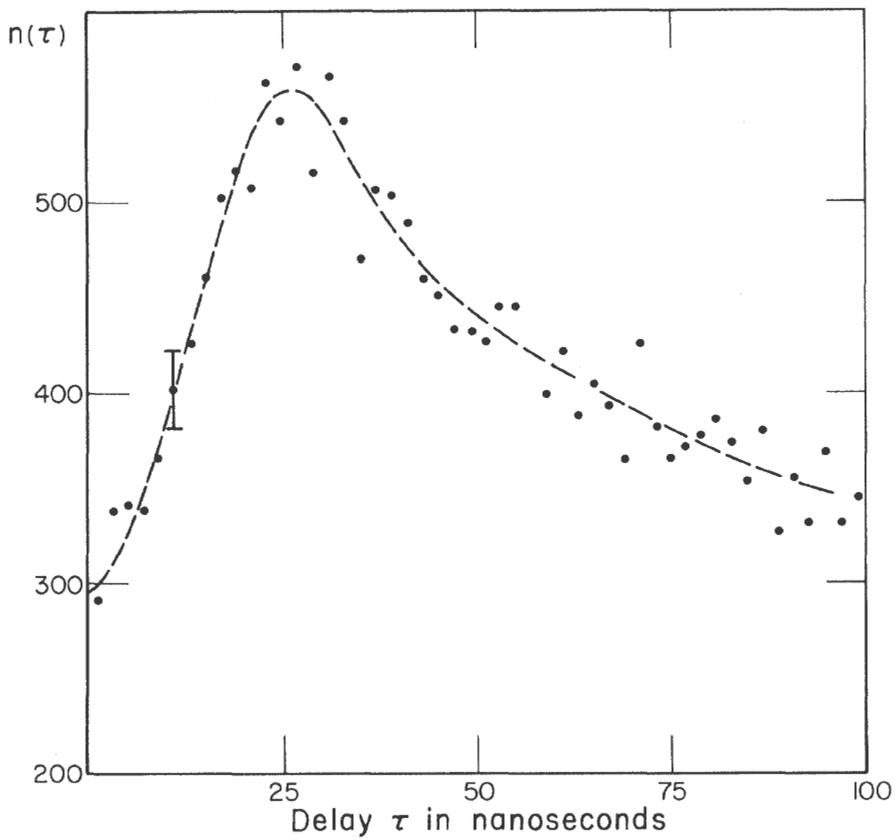
(Received 22 July 1977)



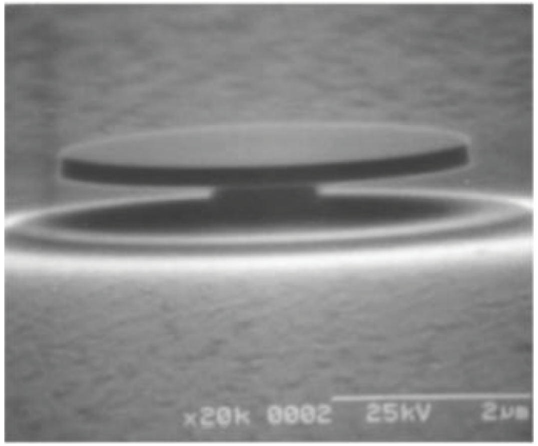


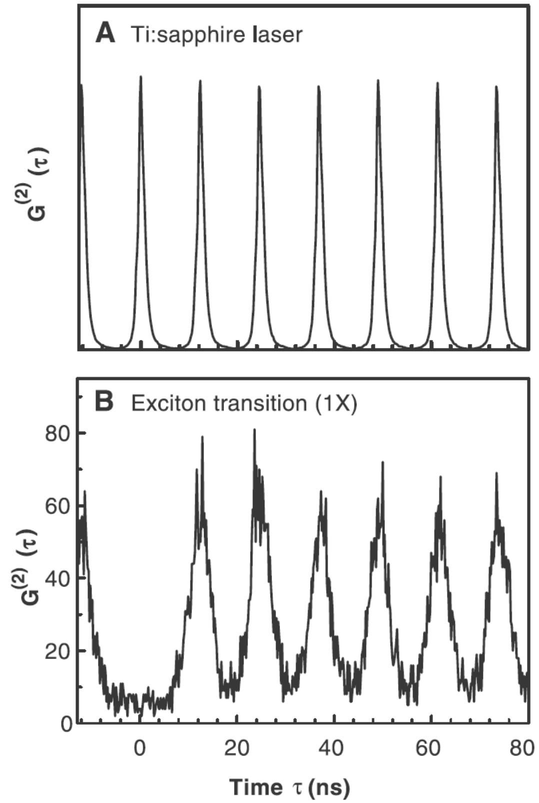
Experiments on single photons

A Quantum Dot Single-Photon Turnstile Device

P. Michler,[1*] A. Kiraz,[1] C. Becher,[1] W. V. Schoenfeld,[2]
P. M. Petroff,[1,2] Lidong Zhang,[1] E. Hu,[1,2] A. Imamoğlu[1,3,4†]

22 DECEMBER 2000 VOL 290 SCIENCE



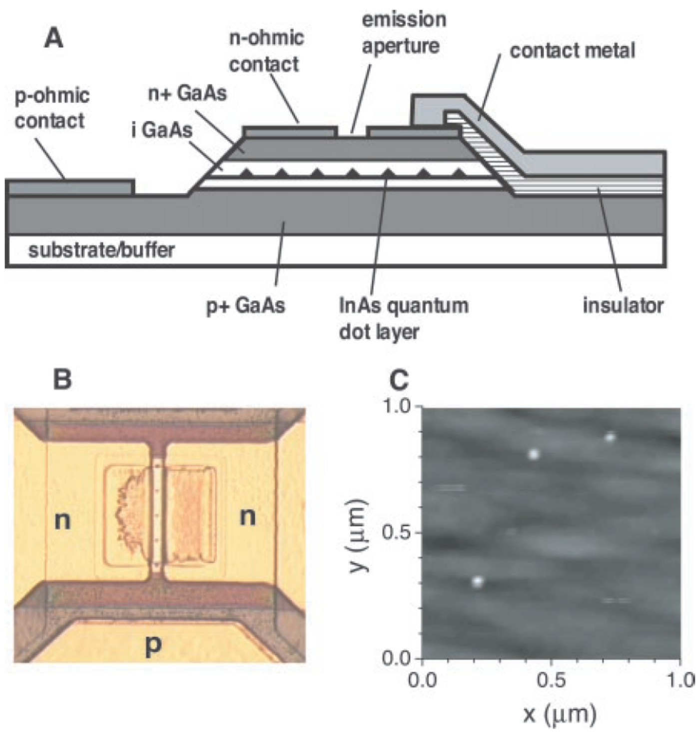


Experiments on single photons II

Electrically Driven Single-Photon Source

Zhiliang Yuan,[1] Beata E. Kardynal,[1] R. Mark Stevenson,[1] Andrew J. Shields,[1*] Charlene J. Lobo,[2] Ken Cooper,[2] Neil S. Beattie,[1,2] David A. Ritchie,[2] Michael Pepper[1,2]

4 JANUARY 2002 VOL 295 SCIENCE



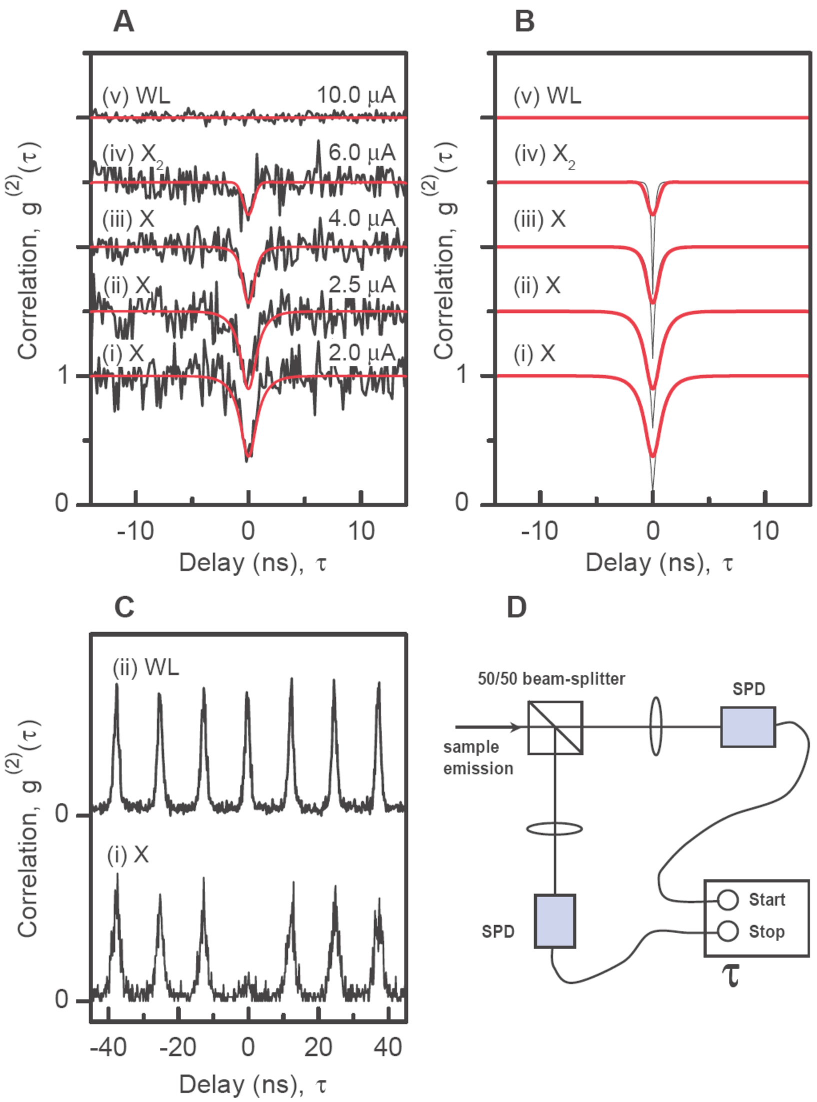


Fig. 3. Experimental proof of photon antibunching and single-photon emission in the electroluminescence of the device. (**A**) Second-order correlation function $g^{(2)}(\tau)$ of the electroluminescence of the single exciton line measured for different injection currents of (i) 2.0, (ii) 2.5, and (iii) 4.0 µA, as well as the biexciton line for 6 µA (iv). For comparison, the correlation trace of the wetting layer (WL) electroluminescence (v) is also shown. The curves are offset vertically for clarity. The solid lines are identical to the broadened calculated curves in (B). (**B**) Calculated second-order correlation function (thin black lines) for electron-hole pair injection rates of (i) 0.45, (ii) 0.55, (iii) 1.31, and (iv) 2.00 pairs per nanosecond and after convolution with a Gaussian function to mimic the finite temporal resolution of the measurement system (thick red lines). (v) Second-order correlation function of an emitter with Poissonian statistics. (**C**) Correlation measured for pulsed electrical injection for (i) quantum dot exciton and (ii) wetting layer emission. (**D**) Schematic of the experimental arrangement for correlation measurements.