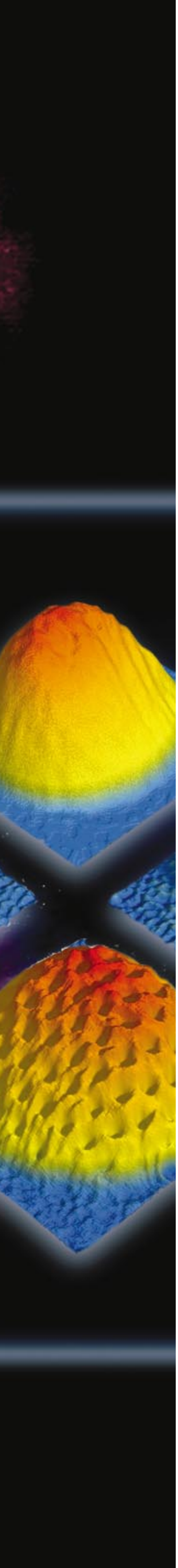

Bose–Einstein condensation of atomic gases

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The early experiments on Bose–Einstein condensation in dilute atomic gases accomplished three long-standing goals. First, cooling of neutral atoms into their motional ground state, thus subjecting them to ultimate control, limited only by Heisenberg's uncertainty relation. Second, creation of a coherent sample of atoms, in which all occupy the same quantum state, and the realization of atom lasers — devices that output coherent matter waves. And third, creation of a gaseous quantum fluid, with properties that are different from the quantum liquids helium-3 and helium-4. The field of Bose–Einstein condensation of atomic gases has continued to progress rapidly, driven by the combination of new experimental techniques and theoretical advances. The family of quantum-degenerate gases has grown, and now includes metastable and fermionic atoms. Condensates have become an ultralow-temperature laboratory for atom optics, collisional physics and many-body physics, encompassing phonons, superfluidity, quantized vortices, Josephson junctions and quantum phase transitions.

The lure of lower temperatures has attracted physicists for much of the past century, and with each advance towards absolute zero, new and rich physics has emerged. Laypeople may wonder why freezing cold is not cold enough; but imagine how many aspects of nature we would miss if we lived on the surface of the Sun. Without inventing refrigerators, we would know only gaseous matter and never observe liquids or solids. Cooling to normal earthly temperatures reveals these dramatically different states of matter, but this is only the beginning: many more states appear with further cooling. The approach into the kelvin range was rewarded with the discovery of superconductivity in 1911 and of superfluidity in ^{4}He in 1938. Cooling into the millikelvin regime revealed superfluidity of ^{3}He in 1972. The advent of laser cooling in the 1980s opened up a new approach to ultralow-temperature physics. Microkelvin samples of dilute atom clouds were generated and used for precision measurements and studies of ultracold collisions. Nanokelvin temperatures were necessary to explore quantum-degenerate gases, such as the Bose–Einstein condensates (BECs) first realized in 1995. Each of these achievements in cooling has been a significant advance, and recognized with a Nobel prize.

The essential techniques for making quantum-degenerate gases are cooling techniques, because at high temperatures a dilute gas of atoms behaves classically. As long as the atoms' de Broglie wavelength $\lambda_{dB} = \hbar/(2Mk_BT)^{1/2}$ is small compared to the spacing between atoms, one can describe their motion with classical trajectories. (λ_{dB} is the position uncertainty associated with the thermal momentum distribution, and increases with decreasing temperature T and atomic mass M.) Quantum degeneracy begins when λ_{dB} and the interatomic distance become comparable. The atomic wave packets overlap, and the gas starts to become a 'quantum soup' of indistinguishable particles. If the atoms are bosons, a condensate — a cloud of atoms all occupying the same quantum state — appears at a precise temperature (which, for an ideal gas, is related to the peak atomic density n by $n\lambda_{dB}^3 = 2.612$). If the atoms are fermions (see Box 1), cooling gradually brings the gas closer to being a 'Fermi sea' in which exactly one atom occupies each low-energy state.

Creating a BEC or a Fermi sea is thus simple in principle — make a gas extremely cold. In most cases, however, quantum degeneracy would simply be pre-empted by the more familiar transitions to a liquid or solid. This more conventional condensation can be avoided only at extremely low densities, about one-hundred-thousandth the density of normal air, so that the formation time of molecules or clusters by three-body collisions (which is proportional to the square of the inverse density) is stretched to seconds or minutes. Because the rate of binary elastic collisions drops only proportionally to the density, these collisions are much more frequent and let the gas equilibrate within about 10 ms, so that degeneracy can be achieved in an effectively metastable gas phase. However, such ultralow density lowers the temperature requirement for quantum degeneracy into the nanokelvin range.

Sub-microkelvin temperatures are reached by combining two procedures. Laser cooling precools the gas so that it can be confined in a magnetic trap[1]. In the second stage — forced evaporative cooling[2] — the trap depth is reduced, allowing the most energetic atoms to escape while the remaining atoms rethermalize at steadily lower temperatures (see ref. 3 for more details). Most experiments with BECs reach quantum degeneracy between 500 nK and 2 μK, at densities between 10^{14} and 10^{15} cm^{-3}. The largest condensates are of 30 million atoms in Na, and a billion in H; the smallest are just a few hundred atoms. Depending on the magnetic trap, the shape of the condensate is either approximately round, with a diameter of 10–50 μm, or cigar-shaped with a diameter about 15 μm and length 300 μm. The full cooling cycle that produces a condensate may take from a few seconds to as long as several minutes.

This review summarizes the recent progress in and beyond Bose–Einstein condensation. Since 1995 this field has grown explosively, drawing researchers from the communities of atomic physics, quantum optics and condensed matter physics. The trapped ultracold vapour has emerged

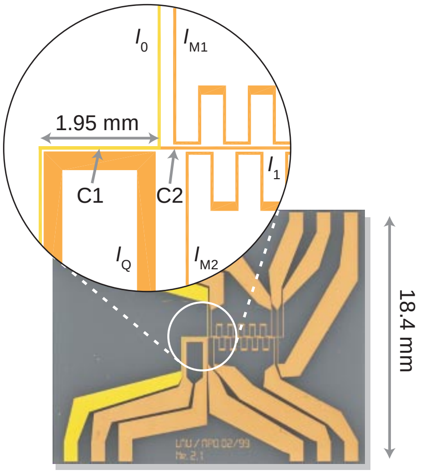


Figure 1 Atom chip. Bose–Einstein condensates (BECs) were created in magnetic traps formed by tiny gold wires that were created lithographically on a substrate[16,17]. The figure shows the pattern used by researchers at the Ludwig-Maximilians University in Munich[17]. Atoms were trapped at positions C1 and C2.

as a new quantum system that is unique in the precision and flexibility with which it can be manipulated. Our field is now at a historic turning point, in which we are moving from studying physics in order to learn about atom cooling to studying cold atoms in order to learn about physics. We begin our review by summarizing new experimental techniques, and then focus on the new ultralow-density condensed matter physics which has been explored.

New techniques and new systems

Evaporative cooling requires a favourable ratio of good to bad collisions — that is, the rate of elastic collisions, which establish thermal equilibrium, must be higher than the rates of inelastic and background gas collisions, which lead to trap loss or molecule formation. A poor collision ratio has hitherto prevented quantum degeneracy being reached in Cs (ref. 4), and until 2000 only ^{87}Rb (ref. 5), ^{23}Na (ref. 6), ^{7}Li (ref. 7) and H (ref. 8) had been Bose-condensed, and most of them only in one specific hyperfine state. But since then, research groups have been able to condense Na (A. Görlitz *et al.*, in preparation) and Li (ref. 9) in both upper and lower hyperfine states, and He (refs 10,11), ^{85}Rb (ref. 12) and K (ref. 13) have been added to the element list. The condensed He atoms are in an excited (but metastable) electronic state, and their internal energy of 20 eV is released when they strike a surface. This process allows their detection with high efficiency, permitting studies of atomic correlations with single-atom counting resolution in analogy with single-photon counting experiments in optics. One might well have expected this internal energy to be released also in binary collisions, making evaporative cooling impossible. An important early theoretical contribution[14] showed this would not in fact occur, and thus gave experimentalists confidence to proceed.

Further flexibility in condensation is offered by the recent development of 'all optical' cooling, in which evaporation is conducted in an optical dipole trap formed of CO_2 laser beams[15], instead of the usual magnetic traps. Reducing the laser intensity allows atoms to escape only at saddle points of the potential, instead of the all-round escape from a magnetic trap during radio-frequency-induced evaporation. But the optical trap holds the gas at a higher density, so that the increased thermalization rate compensates for less efficient evaporation, and Bose–Einstein condensation is quickly reached. Purely optical confinement can be applied to atoms with magnetic moments too small for magnetic trapping, and also offers a route to condensation for atoms that suffer high spin-flipping losses in magnetic traps.

Magnetic traps have also been advanced. They are now offering new capabilities, through miniaturization. Trapping forces are proportional to magnetic field gradients, so shrinking traps onto microchips, using lithographically created arrays of current-carrying wires to generate fields, produces very tight confinement and also reduces power requirements. Several groups are now developing this technology — two have succeeded in Bose-condensing Rb in this kind of environment (Fig. 1)[16,17], while a group at MIT has demonstrated the capability to load a pre-existing condensate into a micro-trap[18]. Further progress in these directions may eventually lead to waveguides and beamsplitters for coherent matter, composing microscopic inertial sensors of unprecedented sensitivity. And it will allow study of quantum fluids in restricted geometries.

While sharply varying magnetic fields can confine atoms into waveguides, smooth background fields can also deliver profound control over atoms, by 'tuning' their collisional properties. At particular field strengths (Feshbach resonances) the energy of a molecular state may be shifted to zero. This allows two colliding atoms to form a temporary bound state, which causes marked changes in their interaction[19]. Such Feshbach resonances were first observed in 1998 (refs 20,21), and have enabled condensation of ^{85}Rb (ref. 12).

Another way to compensate for unfavourable collisional properties of an atomic species is to involve another kind of atom, especially one for which evaporative cooling is very effective, as a refrigerant. Such sympathetic cooling was demonstrated between atoms in the two hyperfine states of ^{87}Rb (ref. 22), the two Rb isotopes[23], and has enabled the condensation of K by cooling it in collisions with Rb atoms[13].

Sympathetic cooling is crucial for cooling degenerate fermions, because the Pauli exclusion principle suppresses collisions among fermions of the same species at low temperatures. Significant Fermi degeneracy was first observed through cooling with two hyperfine states of fermionic K (ref. 24), and has recently been obtained by cooling ^{6}Li with ^{7}Li (refs 9,25) or with Na (ref. 26), or by using two hyperfine states of ^{6}Li (ref. 27; and Box 1 and Fig. 2). So far, no experiments have achieved cooling to less than 25% of the Fermi temperature, and further progress is necessary before we can expect to see any pronounced phenomena in fermionic gases, such as Cooper pairing and superfluidity[28]. The cooling may be limited by Pauli

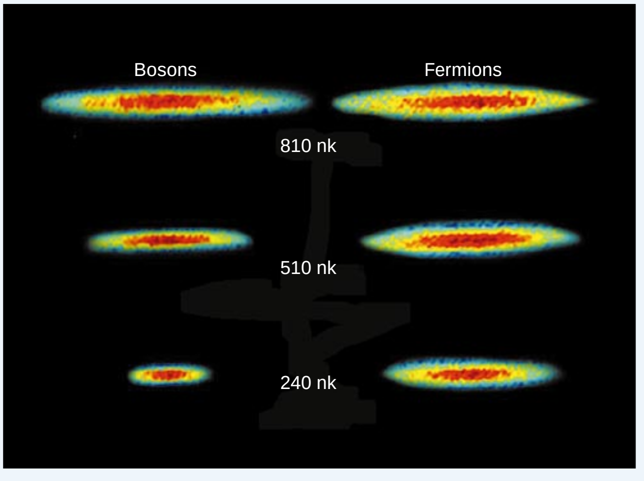


Figure 2 Demonstration of Fermi pressure[25]. The size of the atom clouds in the magnetic trap shrinks as the temperature is reduced by evaporative cooling. Comparison between bosonic ^{7}Li (left) and fermionic ^{6}Li (right) shows the distinctive signature of quantum statistics. The fermionic cloud cannot shrink below a certain size determined by the Pauli exclusion principle. This is the same phenomenon that prevents white dwarf and neutron stars from shrinking into black holes. At the highest temperature, the length of the clouds was about 0.5 mm.

blocking of collisions among fermions, by suppression of collisions as a result of superfluidity when the refrigerant is a BEC[29], or simply by heating[30]. Experimental efforts have just started to investigate these issues.

The family of quantum-degenerate gases is growing rapidly. In addition to optical traps, Feshbach resonances and sympathetic cooling, new techniques such as buffer-gas cooling with cryogenic He gas[31] or cavity cooling[32,33] may extend the ultracold realm to more kinds of atoms, and even to molecules. There is also the prospect of using photoassociation to make molecular condensates from atomic ones: when two atoms collide they can be stimulated into a long-lived molecular state by applying laser beams[34]. There is even speculation that such photoassociative 'superchemistry'[35] might allow coherent cycling of a system between an atomic and a molecular condensate. It is not only the addition of new species, but also progress towards more complex and more refined experiments that now allows the investigation of a remarkable range of physical phenomena.

Ultralow-density condensed matter physics

A condensate is an ultralow-density condensed matter system. Although it is a gas 100,000 times thinner than air, its temperature is so low that even the weak interactions between atoms create effects typical of a 'conventional' condensed system, such as phase transitions, phonons, superfluidity and Josephson oscillations. With multi-component condensates, and confinement in optical lattices, a wide vista opens. The condensed matter physics of ultracold gases is only beginning.

Atoms interact

The first experiments on BECs showed that they were not ideal gases. When more and more atoms were added to a condensate, it swelled beyond the size of the ground state of the trap — a clear sign of repulsive interactions between the atoms[36] — or it collapsed owing to attractive interactions between the atoms[37]. Without these interactions, the BEC would be an ideal gas with properties similar to the photons in the optical laser. The interactions make the BEC a rich, many-body system that displays phenomena such as sound and superfluidity. An attractive feature of Bose–Einstein condensation in dilute atomic gases is that it can be described theoretically from first principles. Therefore, condensates have become a valuable testing ground for the study of interacting many-body systems[38].

The basic theory of the weakly interacting Bose gas was developed from the late 1940s to the early 1960s, and requires that binary collisions are much more frequent than three-body collisions. This condition is fulfilled when the separation between atoms $n^{-1/3}$ is much larger than the *s*-wave scattering length a, that is, the quantity $na^3 \ll 1$ (typically, $na^3 \approx 10^{-6}$). The magnitude of the scattering length gives the effective range of the interatomic forces (typically 1–5 nm for alkali atoms). The stability of large condensates requires repulsive interactions (positive a). For attractive interactions (negative a), the condensate becomes unstable against collapse if it grows above a certain size.

Early theoretical work has led to the Gross–Pitaevskii equation[39,40], which is a wave equation for the macroscopic matter-wave field, and to Bogoliubov's theory of quantum fluctuations around the coherent field[41]. In almost all current experiments, the weakly interacting condition is well fulfilled, and the Gross–Pitaevskii–Bogoliubov theory describes the observed phenomena well. But it has received some modern refinements. The original theories violated the conservation of atoms, and several authors have developed number-conserving formulations[42,43]. The behaviour of condensates at finite temperatures is a frontier of many-body physics[44,45], the experimental exploration of which has so far concentrated mainly on the initial formation of condensates.

Condensate growth

The growth of a condensate is an interesting dynamical process — atoms must find the lowest energy state of the system, and long-range coherence has to be established. Experimentally, this process is observed after fast evaporative cooling, which cools the gas below the transition temperature for Bose–Einstein condensation, but is faster than the growth of the condensate to its equilibrium size[46–48]. A full theoretical description must include the condensate and its elementary excitations, and the interactions with the cloud of thermal atoms (those not part of the condensate). This quantum kinetics problem has been approached from the perspective of quantum optics, which models the condensation process after lasing[49–51]; while condensed matter theorists have investigated the same process in terms of symmetry breaking and phase relaxation[52,53].

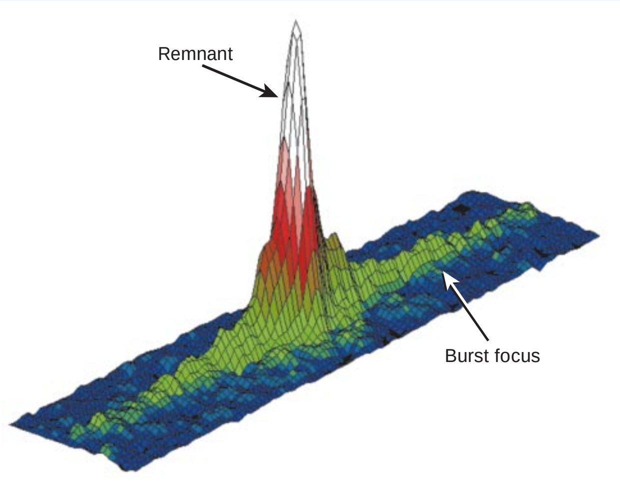


Figure 3 Explosion of a condensate with attractive interactions[55]. By tuning external magnetic fields close to a Feshbach resonance, the interaction between atoms was suddenly switched from zero to attractive. During the collapse, which bears some analogies with a supernova, some atoms were ejected in jets. Because of the magnetic trap, the burst atoms came to a radial focus and were imaged (image size, 60 μm × 310 μm).

Box 1
White dwarfs in the laboratory

When physicists speak in general terms of 'particles', they are actually grouping together two very different kinds of objects, whose distinction becomes manifest at low temperatures. It is possible for arbitrarily many particles in the boson class to occupy a single quantum state, like infinitely gregarious hotel guests that are always willing to crowd into a single room. Particles in the fermion class insist on single occupancy.

Extreme cold effectively reduces the size of the hotel, so that the antisocial nature of fermions becomes important. This manifests itself in Fermi degeneracy pressure, by which a cold cloud of fermionic gas resists being compressed into a smaller volume. This effect keeps white dwarf and neutron stars from collapsing into black holes, and has also recently been observed for ultracold gases in the laboratory (ref. 25; and Fig. 2). Unlike the abrupt phase transition of Bose condensation, the onset of Fermi degeneracy is gradual as temperature drops.

At even lower temperatures, if the effective interaction between fermions is attractive, fermions can bind into pairs, the pairs behaving as bosons. Because electrons and ^{3}He atoms are fermions, this effect is responsible for the condensate-like behaviour of superconductors and superfluid ^{3}He. Producing a paired superfluid in ultracold fermionic vapor is a challenge currently being pursued by several groups.

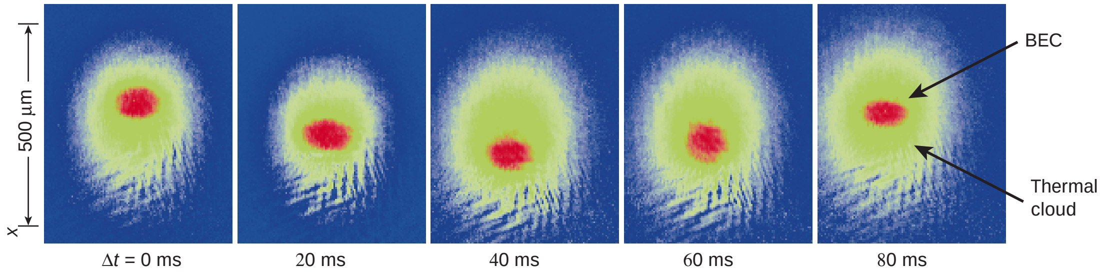


Figure 4 Signature of superfluidity in a Bose-condensed cloud[63]. The condensate and its thermal 'halo' of normal gas respond differently when they are dragged through a periodic potential. In the experiment, the clouds were displaced in a magnetic trap superimposed by an optical lattice. The condensate, distinguished by its much higher density (colour coded in red), tunnelled through the potential peaks and oscillated in the magnetic trapping potential, whereas the normal fraction was pinned by the optical lattice. Interaction between the two clouds eventually led to damping of the condensate motion.

The simplest result is an S-shaped growth curve that reflects initial Bose-stimulated accelerating growth[49]. Observations of the H condensate with its strong two-body losses[48], and of two timescales in the growth of Rb condensates[47], add further richness to this non-equilibrium process.

Special dynamics are associated with condensates with attractive interactions. A collapse of the condensate can be triggered by adding atoms to the condensate[54] or by changing the scattering length through a Feshbach resonance[55]. The observed dynamics of the collapse and subsequent ejection of particles (Fig. 3) are not yet understood theoretically.

Excitations and sound

A normal gas that is as dilute as experimental condensates is in the 'collisionless regime', in which a local perturbation of the density simply diffuses away, unless it extends over a distance longer than the particles' mean free path. [A harmonic trap is special in the sense that it refocuses density fluctuations and transforms a diffusive mode into oscillatory behaviour (but this behaviour is not related to sound).] But in a condensate, collisions that scatter atoms back into the highly occupied quantum state are enormously enhanced, producing a coherent pressure that can support density waves ('zero sound') whose wavelength may be far shorter than the mean free path. Early studies of collective excitations in condensates focused on shape oscillations and their damping. More recently, several nonlinear phenomena have been explored, including coupling between modes[56], soliton propagation[57,58] and quantized vortices (see below).

Superfluidity in a dilute gas

Superfluidity is commonly defined as flow without dissipation. Evidence for superfluidity in liquid He was obtained in 1938, and although superfluidity was almost immediately connected by London to Einstein's theory of Bose–Einstein condensation, it took decades before experimental evidence of a condensate was established with neutron scattering and quantum evaporation[59]. In contrast, in dilute gases condensation was identified first, but it has taken researchers several years to find ways to reveal aspects of superfluidity in these tiny gas clouds.

One way to identify superfluidity is by its characteristic way of breaking down at a precise critical velocity. Only above such a critical velocity is the kinetic energy of the flow sufficient to create excitations. Landau's early theory identified this with the speed of zero sound, because in a flow below this speed phonon production would not be energetically possible. For a macroscopic flow, in all known superfluids, the critical velocity is usually smaller owing to the excitation of vortices.

Critical velocities in BECs were first studied by moving a focused laser beam through the condensate (stirring it)[60,61], and by 'sloshing' the condensate in a corrugated potential[62]. When a magnetic force acted on the condensate and a thermal cloud in such an optical lattice, the condensate moved by coherent tunnelling (Fig. 4), whereas the more energetic thermal cloud was pinned[63]. This counterintuitive behaviour illustrates one of the mysteries of superfluidity and macroscopic quantum mechanics.

Another way to probe a system for superfluid behaviour without 'touching' it is to study torsional modes. If a container of liquid He is

Box 2

Quantized vortices

Quantum mechanics and the wave nature of matter have subtle manifestations when particles have angular momentum, or more generally, when quantum systems are rotating. When a quantum-mechanical particle moves in a circle, the circumference of the orbit has to be an integer multiple of the de Broglie wavelength. This 'quantization rule' leads to the Bohr model and the discrete energy levels of the hydrogen atom. For a rotating superfluid that is described by a macroscopic wavefunction, it leads to the quantization of circulation and quantized vortices. This makes it impossible for a superfluid to rotate as a rigid body — in order to rotate, it must swirl. Vortices are a key feature of superfluid systems.

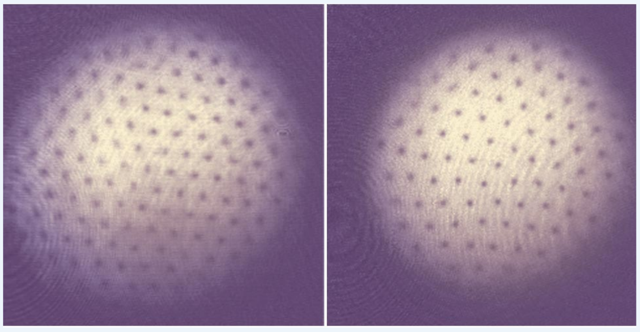


Figure 5 Vortex lattices in rotating BECs[69]. A Na condensate (diameter 60 µm, length 250 µm) was set in rotation by rotating laser beams. It then formed a regular triangular lattice of vortices. Subsequent ballistic expansion resulted in a twenty-times magnification. The images represent two-dimensional cuts through the density distribution and show the density minima due to the vortex cores. The left panel shows a perfect triangular 'Abrikosov' lattice, which, on the right side, has a dislocation. The diameter of the clouds was about 1 mm. (Reprinted with permission from ref. 69. Copyright 2001 American Association for the Advancement of Science.)

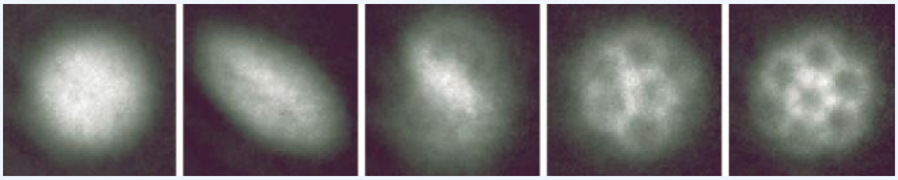


Figure 6 Vortex generation. One route to vortex formation in a BEC is shown in a series of images taken every 150 ms (ref. 75). A condensate was subjected to a rotating anisotropy for 300 ms. An initially small perturbation grew into a strong quadrupolar deformation and finally into a microscopic 'tsunami' that broke into quantized vortices. The field of view in each image is 300 μm. (Copyright 2001 by the American Physical Society.)

slowly rotated, the superfluid fraction does not rotate. Similarly, a condensate held in a yawing trap swings in an irrotational 'scissors mode', the frequency of which is inconsistent with rigid body motion[64,65]. The condition of irrotationality can be violated in a superfluid only by the appearance of quantized vortices (Box 2).

Vortices

It took until 1999 for vortices to be realized experimentally, predominately because some initial failures made other projects seem more attractive. Following a theoretical proposal[66], researchers at the University of Colorado at Boulder constructed a quantized vortex in a two-component condensate by imprinting its phase pattern with laser and radio-frequency fields[67]. A few months later, a group at the Ecole Normale Supérieure in Paris used a rotating laser beam to spin up a condensate, and observed vortex arrays[68]. Similar experiments at MIT, with much larger condensates, produced highly regular triangular lattices of vortices (ref. 69; and Fig. 5). Recently, the Boulder group has formed vortices by cooling a rotating normal gas through the transition for Bose–Einstein condensation[70], and has also managed to create vortex rings[71]. These research teams, together with a group at the University of Oxford[72], have now revealed several aspects of vortex dynamics, including their motion, 'crystallization' into lattices and dissipative escape.

It is perhaps the investigation of the threshold for vortex formation, however, that best illustrates the fruitful interplay between theory and experiment in the field of Bose–Einstein condensation. Vortices in liquid He are usually nucleated at surface roughnesses; but Bose condensates are confined in perfectly smooth 'magnetic containers', and have been stirred with well characterized rotating potentials. Condensates are therefore an ideal testbed for microscopic theories of vortex generation, which attempt to predict the critical rotational velocity above which vortices become stable. Experimentally, vortices are observed only above a critical rotation frequency, whereas in theory, there may be several velocities that are each critical in different senses[73]. Which is relevant?

The Paris group found the critical rotation rate close to the quadrupole shape resonance of the condensate (Fig. 6). Instabilities ('anomalous modes') of vortices already within the condensate were initially proposed to determine this frequency[74]; but collective dynamical instabilities, associated with the resonance in the vortex-free cloud, were later shown to develop into vortices[75,76]. Further numerical and analytical results now indicate that a surface mode version of the Landau theory accurately yields the minimum rotation rate for vortex formation[77,78], and that above this rate the energetic barrier to vortex penetration is also absent, so that tunnelling is not required[79]. Three-dimensional simulations (numerically integrating the Gross–Pitaevskii equation) have begun to probe this complex behaviour (refs 80,81; and Fig. 7).

Multi-component condensates

New physics emerges when different atomic species are mixed and cooled to quantum degeneracy. In the future, mixtures of bosons may allow studies of interpenetrating superfluids. Mixtures of fermions and bosons, as recently realized experimentally[9,25,26], may extend studies of ^{3}He–^{4}He mixtures into new parameter regimes.

Some studies of miscibility, immiscibility and metastability have already been performed using different hyperfine states of Rb or Na (refs 82,83). New phenomena arise when the different components are converted into each other. Atoms can then be in superposition states, and show spin textures, spin waves and coupling between spin and superfluid flow. In addition to quantized vortices, which like ordinary vortices are line-like structures, there can be bubble-like monopoles[84,85]. Whereas vortices and monopoles both require a zero-density core, which is a line or a point, respectively, there can exist textures[86–88] in which the order parameter field twists around in a topologically nontrivial way that does not require a core with vanishing density.

It has also been shown that condensates with spin and antiferromagnetic interactions possess highly correlated singlet ground states[89], which are quite different from the macroscopic wavefunction of simple condensates. But unfortunately this exotic ground state is vulnerable to small magnetic fields, which favour a symmetry-breaking macroscopic wavefunction.

Bose–Einstein condensation in lower dimensions

We have discussed above how the dimensionality of the order parameter space can be raised in multi-component systems. Another opportunity to explore new physics is the reduction of the dimensionality of physical space to two or one dimensions.

To confine a system to lower dimensions usually enhances quantum features and gives rise to new phenomena. A famous example is the quantum Hall effect in a two-dimensional electron gas. Using the tools of atomic physics, condensates can be prepared in a large variety of shapes. Recently, one- and two-dimensional condensates were prepared in highly elongated magnetic and pancake-shaped optical traps[90].

It is a well known theorem that Bose–Einstein condensation cannot occur in systems that are effectively one-dimensional. Yet the practical implications of this theorem are not that significant for cold-atom experiments, which involve finite sizes and numbers of particles, rather than the thermodynamic limit of textbook theory. For an ideal Bose gas of N atoms in a one-dimensional harmonic trap

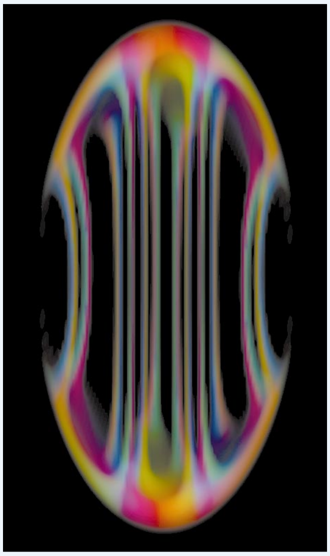


Figure 7 Visualization of vortex lines in a trapped condensate. These results where obtained from numerical integration of the Gross–Pitaevskii mean-field theory in three dimensions[81]. Shown is the condensate density in inverse-rendering mode, that is, low-density regions are depicted bright, and regions beyond the Thomas–Fermi shell have been discarded. As a result, vortex cores within the condensate appear as bright filaments. Colour indicates the phase of the condensate wavefunction, a phase wrap of 2π yielding the quantized circulation of each vortex. (Image courtesy of D. Feder and P. Ketcham, National Institute of Standards and Technology.)

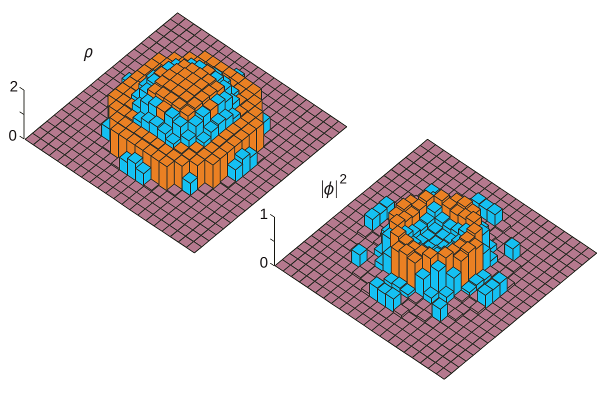


Figure 8 Mott insulator and superfluid phases coexisting in a BEC in a magnetic trap with a superimposed optical lattice[98]. Shown are the total density of cold atoms (left) and the superfluid density (right). Colours are visual guides only. In the insulator phase, the number of atoms shows integer plateaus (left). The superfluid density is maximized between the plateaus. (Copyright 1998 by the American Physical Society.)

Box 3
Phase and density fluctuations

Just as light waves possess both intensity and phase, matter waves have density and phase. In quantum-mechanical terms, density and phase are connected by a Heisenberg uncertainty relation in much the same way as position and momentum; they cannot both be precisely defined at once. Condensates produced by current techniques naturally have phase uncertainties on the order of $N^{-1/2}$, where N is the number of atoms in the condensate.

When two water containers are connected, the water level will be the same in both containers, as any other distribution of the water will cost extra energy. Similarly, when two BECs can exchange particles through a barrier, there are conditions where the repulsive energy between atoms will favour an equal distribution of atoms in which the relative density is more sharply defined than in a 'standard' condensate. Inhibiting atoms from moving between two parts of a cloud thus blurs any difference in phase between the two regions. It is somewhat counterintuitive, but this blurring of relative phase can be used to make atom interferometers more precise, that is, to measure other phases with an accuracy that is better than the so-called 'shot noise' limit.

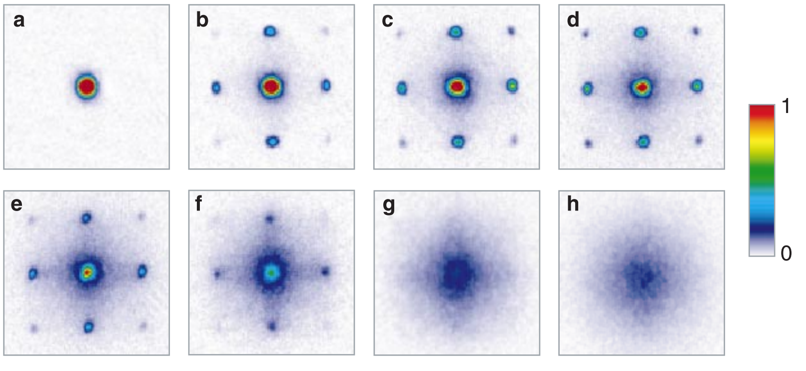
with harmonic frequency ω, below a critical temperature $N\omega/\ln N$ there is a steep increase in population of the ground state, which rises smoothly to approach 100% in a manner similar to Bose–Einstein condensation in a three-dimensional trap[91]. But the effect of interactions in such effectively one-dimensional systems can be strong. Experiments now support theoretical predictions[92] that quasi-condensates, with large phase fluctuations, appear at low temperatures[93]. Another possibility is the realization of a Luttinger liquid[94]. And at extremely low temperatures and densities, wave-mechanical effects make atoms become impenetrable to each other when they propagate in waveguides even if the width of the radial confinement is much larger than the atomic size. This suggests the existence of a transition to a more ordered phase than Bose–Einstein condensation, such as the hard-core gas first analysed by Tonks[95].

Phase versus number
The ideal or weakly interacting condensate represents a classical matter-wave field in the same way as an optical laser emits a classical electromagnetic wave. As in the optical case, where non-classical light has been widely studied, it is possible to create non-classical states of quantum-degenerate matter. Matter-wave fields can exhibit various kinds of squeezing, as quantum corrections to the classical field theory revise the balance between complementary variables (for example, the density and the phase of coherent matter; see Box 3). Interactions between the atoms can be exploited to create entanglement and squeezing and to 'engineer' new non-trivial wavefunctions. This can lead to new forms of quantum matter, and to ensembles that allow higher precision of measurements, for example in atom interferometers[96].

Tunnelling and Josephson junctions
Phase coherence between two separated condensate samples allows the atomic population to oscillate back and forth between them. Individual atoms shuttle back and forth by quantum tunnelling through the energy barrier that separates the samples — this is the classical Josephson effect, which is well described by the Gross–Pitaevskii equation and represents superfluid flow. When the strength of the repulsive interatomic interactions becomes large compared to the tunnelling rate, then the coherent tunnelling ceases abruptly. The increase in energetic costs for density fluctuations leads to a sharply fixed population difference and an indeterminate relative phase (Box 3); because the Josephson current depends on the phase, it ceases.

The onset of number squeezing has been observed in experiments at Yale University, in which a condensate was subjected to a one-dimensional optical-lattice potential, formed by a standing wave of laser light[97]. Increasing laser power raised the barriers between neighbouring potential minima, suppressing tunnelling

Figure 9 Experimental observation of the quantum phase transition from a superfluid to a Mott insulator in a Rb BEC[99]. When the gas was released from the optical lattice the atoms from the more than 100,000 lattice sites overlapped. In the superfluid state, an interference pattern of multiple peaks formed as a result of the phase coherence. When the lattice potential was increased, strong number squeezing in the insulating state suppressed the interference. From **a** to **g**, the lattice potential was increased from zero to twenty recoil energies.



between them, and the number of atoms trapped in each lattice minimum became more sharply defined. When the optical lattice was suddenly turned off, the few dozen mini-condensates overlapped, but with random relative phases, so that interference effects typical of condensates were suppressed.

Quantum phase transition

A more pronounced version of this effect occurs in a three-dimensional lattice, where a quantum phase transition was predicted theoretically from a dilute superfluid to a Mott insulator (ref. 98; and Fig. 8). In the insulator phase, the lattice sites are occupied by the same small number of atoms, generating a highly ordered state. The transport of atoms from site to site is suppressed by an energy gap that results from atom–atom repulsions. This phase transition has been observed recently by researchers at the Ludwig-Maximilians University in Munich, Germany (ref. 99; and Fig. 9). In these experiments the degenerate gas cloud was allowed to equilibrate in the lattice, and was then released. If it expanded from a superfluid phase, strong interference peaks were created; but above a critical lattice strength these abruptly disappeared, indicating the loss of phase coherence between atoms in neighbouring lattice sites. Experiments with cold atoms are thus beginning to create new condensed matter systems, in which a vast range of phenomena may be realized.

Atomic and molecular physics

In this article, we have emphasized BECs as a new system for condensed matter physics, with novel ways to create many-particle wavefunctions. As discussed by Julienne and colleagues elsewhere in this issue (see pages 225–232), the condensate provides a new laboratory also for collision physics at zero energy — the study of the wavefunctions of two and three particles. Over the past few years, intense experimental and theoretical efforts have elucidated atomic interactions and scattering processes near zero energy[100]. Because these interactions depend on the position of individual quantum levels, they can be modified by external magnetic fields through Zeeman shifts[19]. Such Feshbach resonances in atomic collisions are now used to produce 'designer condensates' with adjustable attractive or repulsive interactions.

Condensation is also important as a superior way to create single-particle wavefunctions (occupied with many identical particles): it enlarges the scope of atom optics by providing 'atom lasers'. Condensates are atom sources with high brightness and small divergence and are being used for further advances in atom interferometry and other areas of atom optics, as discussed by Rolston and Phillips in their review on pages 219–224 of this issue. When the interactions of the atoms are involved, atom optics becomes nonlinear and processes such as four-wave mixing and soliton propagation occur, so that the underlying physics overlaps with aspects of condensed matter physics. Previously independent physical disciplines are thus drawn together in the study of BECs.

Outlook

The field of quantum-degenerate gases is at an exciting stage of development. Many of the highlights discussed in this paper have been accomplishments of the past year, and are evidence of the field's continuing rapid development. New atomic systems, in particular ultracold fermions, have considerably broadened the research agenda. Although for bosons the ultimate low-temperature phenomenon — Bose–Einstein condensation — has been accomplished, fermions still pose the challenge of reaching the even lower temperatures that will yield the phase transition into pairing and superfluidity. So the quest for new phenomena at ever lower temperatures will continue. □

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